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# New Atmospheric Turbulence Model for Shuttle Applications

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## **PREFACE**

NASA/Marshall Space Flight Center (MSFC) has developed a new atmospheric turbulence model which is more realistic and less conservative when applied in space shuttle reentry simulations involving engineering calculations of reaction control system fuel expenditures. Both Georgia Institute of Technology and BDM Corporation were contracted to update the required turbulence velocity sigmas and length scales, and to apply them in a white noise filter technique to arrive at a more realistic engineering turbulence model. This model is also envisioned to be useful in other type spacecraft and aircraft simulation studies. This project was funded by the NASA-Johnson Space Center Space Shuttle Office.

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## TECHNICAL MEMORANDUM

### NEW ATMOSPHERIC TURBULENCE MODEL FOR SHUTTLE APPLICATIONS

#### I. INTRODUCTION

It has been determined at NASA [1] that the currently used atmospheric turbulence (wind) model [2,3] for space shuttle reentry simulation is overly conservative. Use of this model in shuttle reaction engine fuel usage calculations assumes severe turbulence all the way from reentry to landing. Johnson Space Center (JSC) sets reaction control system redlines based on these fuel predictions. However, in reality, the orbiter generally returns from space with approximately 270 kg (600 lb) of extra, unused fuel aboard creating an unneeded weight excess. Turbulence in the real atmosphere is patchy or intermittent with quiescent zones present. Therefore, the Environmental Analysis Branch of Marshall Space Flight Center's (MSFC) Earth Science and Applications Division developed a more realistic engineering-oriented turbulence model that can now be used in shuttle simulation work to select a more rational fuel reserve redline value, along with other potential atmospheric turbulence applications.

This modeling task was accomplished in two parts. First, Dr. C.G. Justus of the Georgia Institute of Technology updated the statistical turbulence data base by a literature search to arrive at better estimates of anisotropic horizontal and vertical turbulence velocity standard deviations ( $\sigma_u$  and  $\sigma_w$ ), and length scale parameters ( $L_x$  and  $L_z$ ), from near-surface to 200-km altitude. The y-component ( $v$ ) was not explicitly calculated, but is generated identically to the z-component. These model statistics are available in the form of a program subroutine to evaluate turbulence  $\sigma$ 's and  $L$ 's as a function of altitude. This task is fully documented as part II of this report, and was taken completely from the Georgia Tech final report [4] and reprinted in its entirety here. The results from this task have also been presented at a recent technical conference [5].

The second part of this task was done by Dr. C.W. Campbell and M.K. Doubleday of BDM Corporation who applied and modeled the new turbulence statistics of Justus [4] in a procedure which inputs Gaussian white noise into a low-pass linear filter to output the simulated turbulence in a Gaussian time series. The transfer function of the filter was selected to produce a desired von Karman spectrum, with a more realistic probability distribution. In Campbell's study, for longitudinal spectra, transfer function approximations to the von Karman transfer function, up to fifth order, were derived versus differing sampling rates. The corresponding transverse transfer function is one order higher. The resulting longitudinal and transverse equations used can be directly coded into the shuttle reentry simulation, or into any other type of vehicle flight simulation procedure. The only inputs to the equations are the appropriate turbulence length scale and relative wind velocity turbulent intensity and the sampling rate. Campbell's work is documented in the BDM final report [6] which is also presented in this report as part III. This work has also been presented at technical conferences [7,8]. Larry McWhorter at NASA/JSC is currently implementing this new turbulence model in his shuttle reentry fuel simulation work.

## REFERENCES

1. McWhorter, Larry: private communications.
2. Fichtl, G.H.: "A Technique for Simulating Turbulence for Aerospace Vehicle Flight Simulation Studies." NASA TM-78141, November 1977.
3. NSTS 07700, Vol. X, "Natural Environment Design Requirements," Appendix 10.10, 1975.
4. Justus, C.G.: "A Turbulence Simulation Model for Space Shuttle Reentry Applications." Final Consulting Report for USRA/MSFC, May 1988.
5. Justus, C.G.: "New Height Dependent Magnitudes and Scales for Turbulence Modeling," AIAA paper No. 89-0788, presented at the 27th Aerospace Sciences Meeting, January 9-12, 1989, Reno, Nevada, 9 pp.
6. Campbell, C.W. and Doubleday, M.K.: "Monte Carlo Turbulence Simulation for Shuttle Reentry Studies." BDM Corporation Final Report (BDM/HTV-88-00143-TR-R1) for USRA/MSFC, May 31, 1988.
7. Campbell, C., Doubleday, M., and McWhorter, L.: "Improved Monte Carlo Turbulence Simulation for Shuttle Reentry Studies," AIAA paper No. 89-0789, presented at the 27th Aerospace Sciences Meeting, January 9-12, 1989, Reno, Nevada, 10 pp.
8. Campbell, C.W., Doubleday, M.K., and Johnson, D.: "Monte Carlo Turbulence Simulation for Shuttle Reentry Studies," AMS paper presented at the Third International Conference on the Aviation Weather System, January 30-February 3, 1989, Anaheim, California, pp. 331-334.



## II. UPDATED TURBULENCE STATISTICS AND SUBROUTINE

### A. BACKGROUND

As evidenced by the information in Table 1, from a review of turbulence models by Moorhouse and Heffley (1986), there are a wide variety of techniques which have been used in turbulence simulation. The purpose of this study was to develop a turbulence model for use in space shuttle reentry simulations, which will be simple to use, computationally fast, and consistent with techniques of Monte Carlo modeling and digital filtering, being developed by other researchers for NASA (Campbell and Fichtl, 1985; Campbell, 1986).

Previous NASA methods for simulation of turbulence for aerospace vehicle flight simulations include the turbulence simulation technique of Fichtl (1977) and the turbulence criteria and model presented in Section 2.4 of Turner and Hill (1982). The Fichtl approach was used in Appendix 10.10 of "Natural Environment Design Requirements for the Space Shuttle" (NASA, 1975), and formed the basis for the shuttle simulation turbulence tapes of Tatom and Smith (1982). The Turner and Hill approach has also been adopted and recommended as the turbulence design criteria for other NASA projects (e.g., Adelfang, 1987).

The turbulence model proposed here is based on these techniques, with updates and modifications. The method of Turner and Hill is based on a model probability distribution  $p(\sigma)$  given by:

$$p(\sigma) = \sqrt{2/\pi} (P_1/b_1) \exp \left[ -\frac{\sigma^2}{2b_1^2} \right] + \sqrt{2/\pi} (P_2/b_2) \exp \left[ -\frac{\sigma^2}{2b_2^2} \right], \quad (1)$$

(their equation 2.38), where  $b_1$  and  $b_2$  are the standard deviations of rms gust velocity in nonstorm and storm turbulence, and  $P_1$  and  $P_2$  are the fractions of flight time or distance flown in nonstorm and storm turbulence. Equation (1) assumes a fraction  $P_0$  for flight time or distance in smooth air, such that

$$P_0 + P_1 + P_2 = 1 \quad (2)$$

It should be noted that, for consistency with equation (2), equation (1) should have an added term  $P_0\delta(\sigma)$ , where  $\delta$  is the Dirac delta function.

## B. BASIS OF THE REVISED MODEL

There are several changes in the form of the model developed. The half-Gaussian distribution of equation (1) has the unrealistic feature that the most probable value (the mode) of  $\sigma$  in the  $p(\sigma)$  distribution is  $\sigma = 0$ . The alternate form of distribution suggested is the Rayleigh, which has a more probable value (mode) closer to the mean (expected) value of the distribution. The Rayleigh distribution is given by

$$p(\sigma) = (\sigma/b^2) \exp [-(\sigma/b)^2/2] , \quad (3)$$

which has a mean value  $\bar{\sigma} = (\pi/2)^{1/2}b$ , a mode of  $b$ , and a standard deviation of  $[2 - \pi/2]^{1/2}b$ . For the cumulative probability  $p(\sigma \geq \sigma_1)$ , the Rayleigh distribution produces a Gaussian distribution

$$p(\sigma \geq \sigma_1) = \exp[-(\sigma_1/b)^2/2] . \quad (4)$$

For implementation in the Monte Carlo series simulation (Campbell and Fichtl, 1985; Campbell, 1986), the values of  $\sigma$  can be evaluated from Gaussian-distributed components,  $\sigma_i$  and  $\sigma_j$  (not to be confused with spatial vector components) by the relation  $\sigma = (\sigma_i^2 + \sigma_j^2)^{1/2}$ , where  $\sigma_i$  and  $\sigma_j$  are each selected from a Gaussian (normal) distribution, that is

$$p(\sigma_i) = (2\pi b^2)^{-1/2} \exp[-(\sigma_i/b)^2/2] , \quad (5)$$

and similarly for  $p(\sigma_j)$ .

Equation (3) is used separately to evaluate the probability distribution for no turbulence (with  $\sigma = 0$  and probability  $P_0$ ), moderate turbulence (with  $\sigma$  selected from the distribution with  $b = b_1$  and probability  $P_1$ ), and severe turbulence (with  $\sigma$  selected from the distribution with  $b = b_2$  and probability  $P_2$ ). Instead of using these probability distributions additively, as in equation (1), each of these distributions is used separately, according to which severity level of turbulence is being encountered (none, moderate, or severe). A turbulence severity parameter  $\alpha$  (with value selected from a uniform random distribution between 0 and 1) is used to determine the severity level of the turbulence: there is no turbulence if  $\alpha < P_0$ ; the turbulence is severe if  $\alpha \geq 1 - P_2$ , and moderate otherwise. Minimum vertical depths for layers of moderate and severe turbulence are also specified. Thus, once the series simulation enters a zone with moderate or severe turbulence, it must remain at this severity level until at least the specified minimum depth (or the specified minimum horizontal extent) has been traversed, before it can return to a lower severity level.

The effects of vertical correlations between  $\sigma$  values at one altitude with those at adjacent altitudes are also included. This feature incorporates the fact that rms turbulence gust magnitudes  $\sigma$  must vary more-or-less continuously along the trajectory [not discontinuously as if selected by independent calls upon the probability distribution of equation (3)]. The turbulence  $\sigma$  value changes abruptly, however, when transitioning from one severity layer to another (with only a one-step, linear interpolation smoothing being applied each time a new intensity layer is encountered). No correlations are assumed between sigmas in two layers which are spatially separated by a layer of lower turbulence magnitude. Thus, each time a layer of higher than current turbulence severity is encountered, the random number generator sequence for the sigma selections is reinitialized.

A major portion of the model development project has been a literature survey to develop revised parameter values for the data on turbulence intensities (mean  $\sigma$  values), scales, and probabilities of intensity levels. In addition, new parameter values were required for the vertical scale of the  $\sigma$  interlevel correlation, and the minimum vertical sizes for moderate and severe turbulence layers. Anisotropic horizontal and vertical values are provided for the turbulence intensity and scale parameters.

### C. MODEL DESCRIPTION

The literature survey for updating the turbulence parameter values consisted of a search of the Scientific and Technical Aerospace Abstracts, the International Aerospace Abstracts, and the Meteorological and Geostrophysical Abstracts for the period 1970 to present. Parameter values resulting from this literature review have been incorporated into the turbulence simulation subroutine.

Information on horizontal and vertical scales of turbulence (integral scale in the velocity correlation function) were averaged from data in McCloskey et al. (1971), U.S. Department of Defense (1975), Fichtl (1977), Hasty (1977), Justus et al. (1980), Turner and Hill (1982), Frost et al. (1985), Murrow (1986), and Reid and Vincent (1987). Resulting average values for these horizontal and vertical scales are shown in Figure 1 as a function of altitude from the surface to 200 km.

Values of the sigma components  $\sigma_i$  and  $\sigma_j$  to be selected from the Rayleigh distribution by equation (4) are assumed to be correlated over horizontal and vertical separations with scale values (integral scale) which are related to the horizontal and vertical scales of the turbulent velocity, through a ratio which was assumed to vary from 10.0 at the surface to 5.0 at 20 km and higher. For example, with a horizontal turbulence velocity scale of 0.52 km at the surface, the horizontal sigma scale is assumed to be 5.2 km. Assumptions such as these were required because no observational data were found to provide direct estimates of the sigma scales. These ratio values (5–10) were assumed since turbulent velocity statistics cannot accurately be determined from measurements unless the turbulence is stationary (relatively constant sigma) over at least 5–10 velocity scale values; hence a presumption that the sigma scales relative to the velocity scales are of at least this magnitude range (with larger ratio values anticipated to occur at lower altitudes).

For data on the average value  $\bar{\sigma}_u$  (the average longitudinal turbulence intensity), data were averaged over information taken from Pershikov (1969), McCloskey et al. (1971), Ryan et al. (1971), Fichtl (1977), Kao et al. (1977), Waco (1978), Justus et al. (1980), Vinnichenko et al. (1980), Moorhouse and Woodcock (1982), Murrow et al. (1982), Turner and Hill (1982), Hocking (1983), Frost et al. (1985), Hill (1986), Murrow (1986), and Andrews et al. (1987). For the Fichtl (1977) values of  $\sigma_u$ , a weight value of 0.6 was used, since these values were taken to represent extreme values rather than average values.

Data on the average value  $\bar{\sigma}_w$  (or on the ratio of  $\bar{\sigma}_w/\bar{\sigma}_u$ ) were taken from McCloskey et al. (1971), Ryan et al. (1971), U.S. Department of Defense (1975), Fichtl (1977), Waco (1978), Vinnichenko et al. (1980), Murrow et al. (1982), Turner and Hill (1982), Frost et al. (1985), Sato (1985), and Murrow (1986). All of these data were interpreted in terms of an average ratio for  $\sigma_w/\bar{\sigma}_u$ , which was then multiplied by the average  $\bar{\sigma}_u$ , determined above, to arrive at the final value for the average  $\bar{\sigma}_w$ .

Values of the ratio ( $\gamma = \bar{\sigma}_2/\bar{\sigma}_1$ ) of the magnitude of severe turbulence ( $\bar{\sigma}_2$ ), relative to the magnitude of moderate turbulence ( $\bar{\sigma}_1$ ), were averaged from data in Kao et al. (1977), Moorhouse and Woodcock (1982), and Turner and Hill (1982). No values for the  $\gamma$  intensity ratio were found for altitudes above 18 km; therefore a value of  $\bar{\sigma}_2$  equal to 1.5 times  $\bar{\sigma}_1$  was assumed at 30 km and higher, with a smooth transition from the value of  $\gamma = 1.84$  at 18 km to  $\gamma = 1.5$  at 30 km.

Values of the total probability  $P = P_1 + P_2$  for encountering turbulence (either moderate or severe) were taken and averaged from McCloskey et al. (1971), Wilson et al. (1971), Ehrenberger (1975), Waco (1976), Hasty (1977), Zimmerman and Murphy (1977), Turner and Hill (1982), and Ehrenberger (1987). Values for  $\rho = P_2/P_1$ , the ratio of the probability of encountering severe turbulence to that for encountering moderate turbulence, were obtained and averaged from McCloskey et al. (1971), Waco (1976), and Turner and Hill (1982). No values of  $\rho$  were found in the literature for altitudes above 30 km (where  $\rho = 0.1$ ). A steady decrease to a value of  $\rho = 0.01$  was assumed at 120 km, with  $\rho = 0.01$  between 120 and 200 km. From values of the total probability  $P$ , and the ratio  $\rho$ , separate values of  $P_1$  and  $P_2$  were obtained from

$$P_1 = P/(1 + \rho) \quad \text{and} \quad P_2 = \rho P/(1 + \rho) \quad . \quad (6)$$

From values of  $\rho$  and the severity magnitude ratio  $\gamma$ , separate values for average sigma for moderate turbulence ( $\bar{\sigma}_1$ ) and for severe turbulence ( $\bar{\sigma}_2$ ) can be obtained from the overall average sigma  $\bar{\sigma}$  for both horizontal and vertical components (u and w) via

$$\bar{\sigma}_{1u} = \frac{(1 + \rho)\bar{\sigma}_u}{(1 + \rho\gamma)} \quad \text{and} \quad \bar{\sigma}_{2u} = \frac{\gamma(1 + \rho)\bar{\sigma}_u}{(1 + \rho\gamma)} \quad , \quad (7)$$

$$\bar{\sigma}_{1w} = \frac{(1 + \rho)\bar{\sigma}_w}{(1 + \rho\gamma)} \quad \text{and} \quad \bar{\sigma}_{2w} = \frac{\gamma(1 + \rho)\bar{\sigma}_w}{(1 + \rho\gamma)} \quad . \quad (8)$$

Values for the resulting averages  $\bar{\sigma}_{2u}$  and  $\bar{\sigma}_{2w}$  as a function of altitude for severe turbulence are shown in Figure 2. The profiles of  $\bar{\sigma}_{1u}$  and  $\bar{\sigma}_{1w}$  values for moderate turbulence are shown in Figure 3. Values of the average sigmas for composite turbulence,  $\bar{\sigma}_c$ , defined to be probability-weighted values of  $\bar{\sigma}_1$  and  $\bar{\sigma}_2$ , are defined by

$$\bar{\sigma}_c = P_0 \cdot (0) + P_1 \bar{\sigma}_1 + P_2 \bar{\sigma}_2 \quad (9)$$

for both u and w components [where the normalization  $P_0 + P_1 + P_2 = 1$  has been used, from equation (2)]. The resulting composite turbulence  $\bar{\sigma}_c$  values are shown in Figure 4, as functions of height, for both horizontal and vertical components.

Up to about 20 km, the magnitudes (Figures 2–4) and scales (Figure 1) for the turbulence model are taken to represent atmospheric turbulence as conventionally defined. Above this height, the magnitudes and scales are taken to represent either gravity waves (which in high-speed flight may produce the same dynamical effects on the vehicle as does turbulence), or the turbulence which may result from gravity wave breaking or gravity wave dissipation at critical levels.

For the minimum vertical extent of turbulence layers, data values from Ehrenberger (1975), Röttger (1980), Barat (1982), Tanaka and Yamanaka (1984), Yamanaka and Tanaka (1984), Cot and Barat (1986), and Ehrenberger (1987) were averaged. No data on vertical layer thickness were available above the 60–90 km layer, so an extrapolated increase rate in the vertical layer size above this altitude range was assumed.

Horizontal minimum extent of turbulence layers was derived from average data values taken from Ehrenberger (1975), Murrow et al. (1982), Hasty (1977), and Waco (1978). No data were available above 25 km, so an extrapolated height increase for the horizontal layer size was assumed above this height. No clear data were available on the relative sizes for severe turbulence relative to the size for moderate turbulence layers. Therefore the minimum vertical and horizontal sizes for severe turbulence layers were taken to be 1/2 that for the moderate layers.

The attached program listing shows the model implementation as subroutine TURBSIG, which is to be used for simulating turbulence intensity values ( $\sigma$ 's) at a given altitude  $z$  ( $0 \leq z \leq 200$  km). The subroutine returns values for  $\sigma_u$  and  $\sigma_w$ , the horizontal and vertical sigma components (in m/s), and  $L_x$  and  $L_z$ , the horizontal and vertical integral scale values (in km).

Several model options (specified by the input parameter MODEL) are available: MODEL = 0 for the complete model with all specified parameters (probabilities  $P_0$ ,  $P_1$ ,  $P_2$ , average sigma values,  $\bar{\sigma}_{1u}$ ,  $\bar{\sigma}_{2u}$ ,  $\bar{\sigma}_{1w}$ ,  $\bar{\sigma}_{2w}$ , scales  $L_x$ ,  $L_z$ , etc.) with the local, spatially dependent, sigma values selected from the appropriate Rayleigh distribution for moderate and severe turbulence intensity levels; MODEL = 1 for moderate turbulence throughout the whole run, with sigma values selected from the Rayleigh distribution which has  $\bar{\sigma} = \bar{\sigma}_1$ ; MODEL = 2 for severe turbulence throughout the whole run with sigma values selected from the Rayleigh distribution which has  $\bar{\sigma} = \bar{\sigma}_2$ ; MODEL = 3 for composite turbulence (probability-weighted average of severe, moderate, and non-turbulent) throughout the whole run, with sigmas selected from the Rayleigh distribution which

has  $\bar{\sigma} = \bar{\sigma}_c$ ; MODEL = 4 for no turbulence throughout the whole run ( $\sigma \equiv 0$ ); MODEL = 5 for moderate turbulence with  $\sigma = \bar{\sigma}_1$  (no Rayleigh distribution) throughout the whole run; MODEL = 6 for severe turbulence with  $\sigma = \bar{\sigma}_2$  throughout the whole run (no Rayleigh distribution); and MODEL = 7 for composite turbulence with  $\sigma = \bar{\sigma}_c$  throughout the whole run (no Rayleigh distribution).

The main program TESTSIG is designed to be used for producing test output of the TURBSIG subroutine, and illustrates how TURBSIG is to be used in the reentry trajectory simulation programs. The user must select a starting random number (any odd positive integer) for the random number generator (RAND), and a value of MODEL must be selected. The TURBSIG subroutine must also be called once with displacement values  $dx = 0$  and  $dz = 0$ , in order to initialize all of its variables. On subsequent calls, the trajectory program must pass to the subroutine the displacement values ( $dx$  and  $dz$ , in km) of current position from previous position. Only the magnitudes of the displacements are of importance, so positive or negative values may be used.

## REFERENCES

- Adelfang, S.I. (1987): "Wind and Turbulence Data for Development of Design Criteria for Unmanned High-Altitude Low-Speed Endurance Aircraft," NASA CR-4065 (N87-20699).
- Andrews, D.G., J.R. Holton, and C.B. Leovy (1987): *Middle Atmosphere Dynamics*. Academic Press, New York, Figure 19, p. 16.
- Anonymous (1974): "Characteristics of Atmospheric Turbulence Near the Ground, Part III: Variations in Space and Time in Strong Winds (Neutral Atmosphere)," Engineering Sciences Data Unit Item No. 74031, London, England.
- Anonymous (1975): "Characteristics of Atmospheric Turbulence Near the Ground, Part III: Variations in Space and Time in Strong Winds (Neutral Atmosphere)," Engineering Sciences Data Unit Item No. 75001, London, England.
- Barat, J. (1982): "Some Characteristics of Clear-Air Turbulence in the Middle Stratosphere," *J. Atmos. Sci.*, 39, 2553-2564.
- Barr, N.M. et al. (1974): "Wind Models for Flight Simulator Certification of Landing and Approach Guidance and Control Systems," FAA-RD-74-206.
- Campbell, C.W. and G.H. Fichtl (1985): "Recent Advances in Monte Carlo Turbulence Simulations," AMS Conference on Aerospace and Range Meteorology, August 27-29, Huntsville, AL.
- Campbell, C.W. and V.A. Sandborne (1985): "A Spatial Model of Wind Shear and Turbulence," *J. Aircraft*, 21(12), 929-925.
- Campbell, C.W. (1986): "Monte Carlo Turbulence Simulation Using Rational Approximations to Von Karman Spectra," *AIAA Journal*, 24(1), 62-66.
- Chalk, C.R. et al. (1969): "Background Information and User Guide for MIL-F-8785B(ASG), Military Specification for Flying Qualities of Piloted Airplanes," AFFDL-TR-69-72.
- Cot, C. and J. Barat (1986): "Wave-Turbulence Interaction in the Stratosphere - A Case Study," *J. Geophys. Res.*, 91, 2749-2756.
- Dryden, H.L. (1961): "A Review of the Statistical Theory of Turbulence," *Turbulence - Classic Papers on Statistical Theory* (S.K. Friedlander and L. Topper, eds.), Interscience Publishers, Inc., New York.
- Durand, T.S. (1967): "Carrier Landing Analyses," Systems Technology, Inc., Technical Report No. 137-2.

- Ehernberger, L.J. (1975): "High Altitude Gust Acceleration Environment as Experienced by a Supersonic Airplane," NASA TN D-7868 (N75-13791).
- Ehernberger, L.J. (1987): "High Altitude Turbulence for Supersonic Cruise Vehicles," NASA TM-88285.
- Etkin, B. (1959a): Dynamics of Flight, Stability and Control. John Wiley and Sons, Inc., New York.
- Etkin, B. (1959b): "Theory of the Response of Airplanes to Random Atmospheric Turbulence," J. Aero/Space Sciences, 409-420.
- Etkin, B. (1961): "Theory of the Flight of Airplanes in Isotropic Turbulence - Review and Extension," AGARD Report 372.
- Etkin, B. (1972): Dynamics of Atmospheric Flight. John Wiley and Sons, Inc., New York.
- Etkin, B. (1980): "The Turbulent Wind and Its Effect on Flight," AIAA 80-1836.
- Fichtl, G.H. (1977): "A Technique for Simulating Turbulence for Aerospace Vehicle Flight Simulation Studies," NASA TM-78141.
- Fortenbaugh, R.L. (1978): "Mathematical Models for the Aircraft Operational Environment of DD-963 Class Ships," Vought Corporation Report No. 2-55800/8R-3500.
- Foy, W.H. and W.B. Gartner (1979): "Piloted Flight Simulation Study of Low-Level Wind Shear, Phase 4," FAA-RD-79-84.
- Frost, W. and D.W. Camp (1977): "Wind Shear Modeling for Aircraft Hazard Definition," FAA-RD-77-36.
- Frost, W. et al. (1985): "Analysis of Data from NASA B-57B Gust Gradient Program," NASA CR-178736 (N86-23604).
- Gaonkar, G.H. (1980): "Review of Non-Stationary Gust-Responses of Flight Vehicles," AIAA 80-0703.
- Hasty, P.L. (1977): "A Description of the Atmospheric Turbulence Environment Derived From the Critical Atmospheric Turbulence (ALLCAT) Program," AFFDL-TR-77-4 (AD-A42-549).
- Heffley, R.K. and W.F. Jewell (1978): "Study of a Safety Margin System for Powered-Lift STOL Aircraft," NASA CR-152139.
- Hill, C.K. (1986): "Analysis of Jimsphere Pairs for Use in Assessing Space Vehicle Ascent Capability," NASA TP-2573.



- Hocking, W.K. (1983): "Atmospheric Turbulence Intensities Measured with an HF Radar at 35°S, Part 2," J. Atmos. Terr. Phys., 45, 103-114.
- Hoh, R.H. and W.F. Jewell (1976): "Investigation of the Vulnerability of Powered Lift STOLs to Wind Shear," NASA CR-152064.
- Hoh, R.H. et al. (1982): "Proposed MIL Standard and Handbook – Flying Qualities of Air Vehicles," AFWAL-TR-82-3081.
- Holly, W.E. and A.E. Bryson, Jr. (1975): "Wind Modeling and Lateral Aircraft Control for Automatic Landing," Stanford University, Dept. of Aeronautics and Astronautics, SUDAAR No. 489.
- Houbolt, J.C. (1973): "Atmospheric Turbulence," AIAA Journal, 11(4), 421-437.
- Jacobson, I.D. and D.S. Joshi (1977): "Investigation of the Influence of Simulated Turbulence on Handling Qualities," J. Aircraft, 14(3), 272-275.
- Jansen, C.J. (1977a): "A Digital Turbulence Model for the NLR Moving-Base Flight Simulator, Part I," National Aerospace Laboratory, NLR Memorandum VS-77-024 U.
- Jansen, C.J. (1977b): "A Digital Turbulence Model for the NLR Moving-Base Flight Simulator, Part II," National Aerospace Laboratory, NLR Memorandum VS-77-025 U.
- Jewell, W.F. and R.K. Heffley (1978): "A Study of Key Features of the RAE Atmospheric Turbulence Model," NASA CR-152194.
- Jewell, W.F. et al. (1979): "Powered-Lift Aircraft Handling Qualities in the Presence of Naturally-Occurring and Computer-Generated Atmospheric Disturbances," FAA-RD-79-59.
- Johnson, W.A. and G.L. Teper (1974): "Analysis of Vortex Wake Encounter Upsets," NASA CR-127491.
- Jones, J.G. (1976): "Modeling of Gusts and Wind Shear for Aircraft Assessment and Certification," Royal Aircraft Establishment, CAARC Symposium on Operational Problems, India.
- Justus, C.G., G.R. Fletcher, F.E. Gramling, and W.B. Pace (1980): "The NASA/MSFC Global Reference Atmospheric Model – MOD 3 (With Spherical Harmonic Wind Model)," NASA CR-3256.
- Kao, S.K. et al. (1977): "Characteristics of the Motions, Turbulence Intensity, Diffusivity, Flux of Momentum and Sensible Heat in the Upper Atmosphere," NASA CR-2782, (N77-17666).
- Lappe, U.O. (1966): "Low-Altitude Turbulence Model for Estimating Gust Loads on Aircraft," J. Aircraft, 3(1).

- McCloskey, J.W. et al. (1971): "Statistical Analysis of LO-LOCAT Turbulence Data for Use in the Development of Revised Gust Criteria," AFFDL-TR-71-29 (AD-731-139).
- Moorhouse, D.J. and R.K. Heffley (1986): "The Status of Military Specifications with Regard to Atmospheric Turbulence," in *Atmospheric Turbulence Relative to Aviation, Missile, and Space Programs*, D.W. Camp and W. Frost, eds., NASA CP-2468, pp. 181-198.
- Moorhouse, D.J. and R.J. Woodcock (1982): "Background Information and User Guide for MIL-F-8785C, Military Specification - Flying Qualities of Piloted Airplanes," AFWAL-TR-81-3109 (AD-A119-421).
- Murrow, H.N. et al. (1982): "Power Spectral Measurements of Clear-Air Turbulence to Long Wavelengths for Altitudes up to 14,000 Meters," NASA TP-1979 (N82-24790).
- Murrow, H.N. (1986): "Measurements of Atmospheric Turbulence," in *Atmospheric Turbulence Relative to Aviation, Missile, and Space Programs*, D.W. Camp and W. Frost, eds., NASA CP-2468, pp. 73-92.
- NASA (1975): "Natural Environment Design Requirements, Appendix 10.10, Space Shuttle Flight and Ground Specifications, Level II, Program Definitions and Requirements," JSC 07700, Vol. X, Rev. 5, NASA Lyndon B. Johnson Space Flight Center, Houston, TX.
- Nave, R.L. (1978): "Development and Analysis of a CVA and a 1052 Class Fast Frigate Air Wake Model," NADC-78182-60.
- Pershikov, V.P. (1969): "Analysis of a Turbulent Wind in the Meteor Zone," *Geomag. Aeron.*, 9, 1114-1116.
- Reeves, P.M. (1969): "A Non-Gaussian Turbulence Simulation," AFFDL-TR-69-67.
- Reeves, P.M. et al. (1974): "Development and Application of a Non-Gaussian Atmospheric Turbulence Model for Use in Flight Simulators," NASA CR-2451.
- Reid, I.M. and R.A. Vincent (1987): "Measurements of the Horizontal Scales and Phase Velocities of Short Period Mesospheric Gravity Waves at Adelaide, Australia," *J. Atmos. Terr. Phys.*, 49, 1033-1048.
- Röttger, J. (1980): "Structure and Dynamics of the Stratosphere and Mesosphere Revealed by VHF Radar Investigations," *PAGEOPH*, 118, 494-527.
- Ryan, J.P. et al. (1971): "Medium Altitude Critical Atmospheric Turbulence (MEDCAT) Data Processing and Analysis," AFFDL-TR-71-82 (AD-732-878).
- Sato, T. (1985): "High-Resolution MST Observations of Turbulence by Using the MU Radar," *Radio Sci.*, 20, 1452-1460.

- Sinclair, S.R.M. and T.C. West (1978): "Handling Qualities of a Simulated STOL Aircraft in Natural and Computer-Generated Turbulence and Shear," Piloted Aircraft Environment Simulation Techniques, AGARD-CP-249.
- Skelton, G.B. (1968): "Investigations of the Effects of Gusts on V/STOL Craft in Transition and Hover," AFFDL-TR-68-85.
- Tanaka, H. and M.D. Yamanaka (1984): "Meso- and Microscale Structures of Stratospheric Winds – A Quick Look of Balloon Observation," J. Meteorol. Soc. Japan, 62, 177–182.
- Tatom, F.B. and S.R. Smith (1982): "Advanced Space Shuttle Simulation Model," NASA CR-3541.
- Tomlinson, B.N. (1975): "Developments in the Simulation of Atmospheric Turbulence," Royal Aircraft Establishment, Technical Memorandum FS 46.
- Turner, R.E. and C.K. Hill (1982): "Terrestrial Environment (Climatic) Criteria Guidelines for Use in Aerospace Vehicle Development, 1982 Revision," NASA TM-82473.
- United States Department of Defense (1969): "Flying Qualities of Piloted Aircraft," Military Specification MIL-F-8785B.
- United States Department of Defense (1975): "Military Specification – Flight Control Systems – Design, Installation and Test of Piloted Aircraft, General Specification for," MIL-F-9490D (M302-4212).
- United States Department of Defense (1980): "Flying Qualities of Piloted Aircraft," Military Specification MIL-F-8785C.
- van de Moesdijk, G.A.J. (1978): "Non-Gaussian Structure of the Simulated Turbulent Environment in Piloted Flight Simulation," Delft University of Technology, Department of Aerospace Engineering, Memorandum M-304.
- Vinnichenko, N.K., N.Z. Pinus, S.M. Shmeter, and G.N. Shur (1980): Turbulence in the Free Atmosphere. 2nd edition, Consultants Bureau, New York, 310 pp.
- Von Karman, T. (1961): "Progress in the Statistical Theory of Turbulence," Turbulence – Classic Papers on Statistical Theory (S.K. Friedlander and L. Topper, eds.), Interscience Publishers, Inc., New York.
- Waco, D.E. (1976): "Variation of Turbulence with Altitude to 70,000 ft," J. Aircraft, 13, 981–986.
- Waco, D.E. (1978): "Mesoscale Wind and Temperature Fields Related to an Occurrence of Moderate Turbulence Measured in the Stratosphere Above Death Valley," Mon. Wea. Rev., 106, 850–858.

- Wilson, R.J., et al. (1971): "Evaluation of Effects of High-Altitude Turbulence Encounters on the XB-70 Airplane," NASA TN D-6457 (N71-30718).
- Yamanaka, M.D. and H. Tanaka (1984): "Multiple 'Gust Layers' Observed in the Middle Stratosphere," in Dynamics of the Middle Atmosphere, J.R. Holton and T. Matsuno, eds., D. Reidel Pub. Co., Boston, pp. 117-138.
- Zhu, S. and B. Etkin (1985): "Model of the Wind Field in a Downburst," J. Aircraft, July.
- Zimmerman, S.P. and E.A. Murphy (1977): "Stratospheric and Mesospheric Turbulence," AFGL-TR-78-0020 (AD-A050-128).

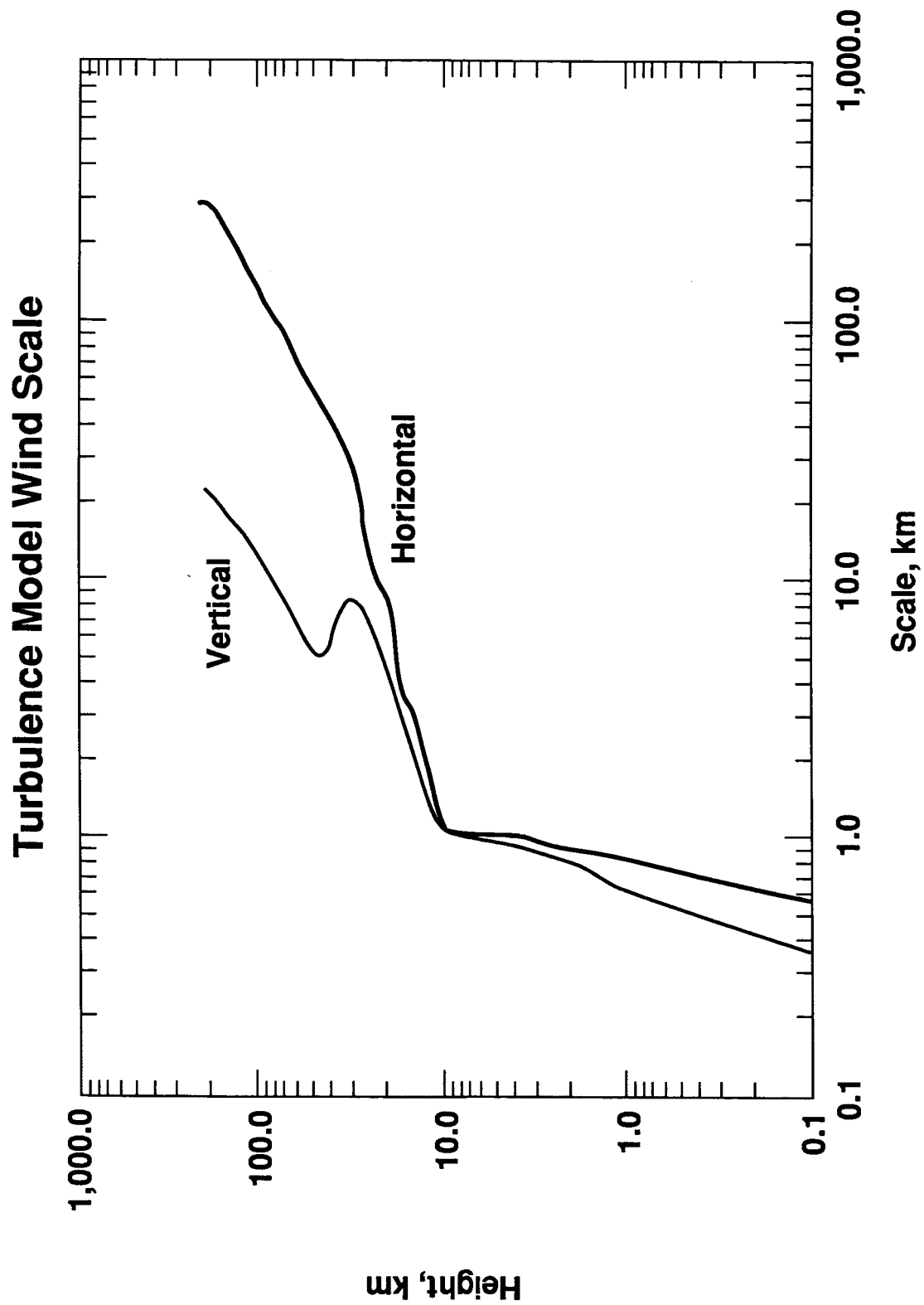


Figure 1. Horizontal and vertical scales (km) for the turbulence model as a function of height.

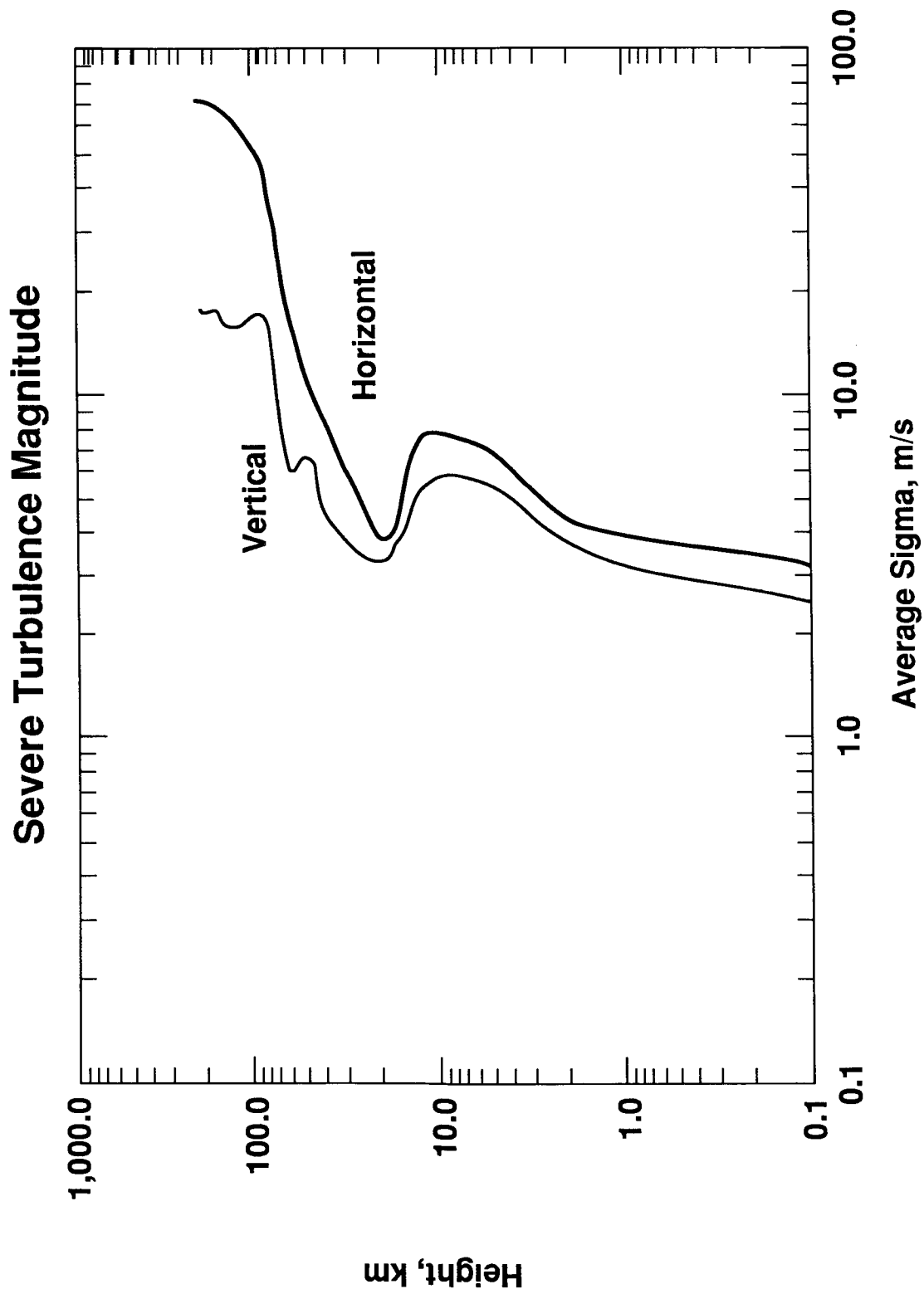


Figure 2. Horizontal and vertical magnitudes for severe turbulence as a function of height.

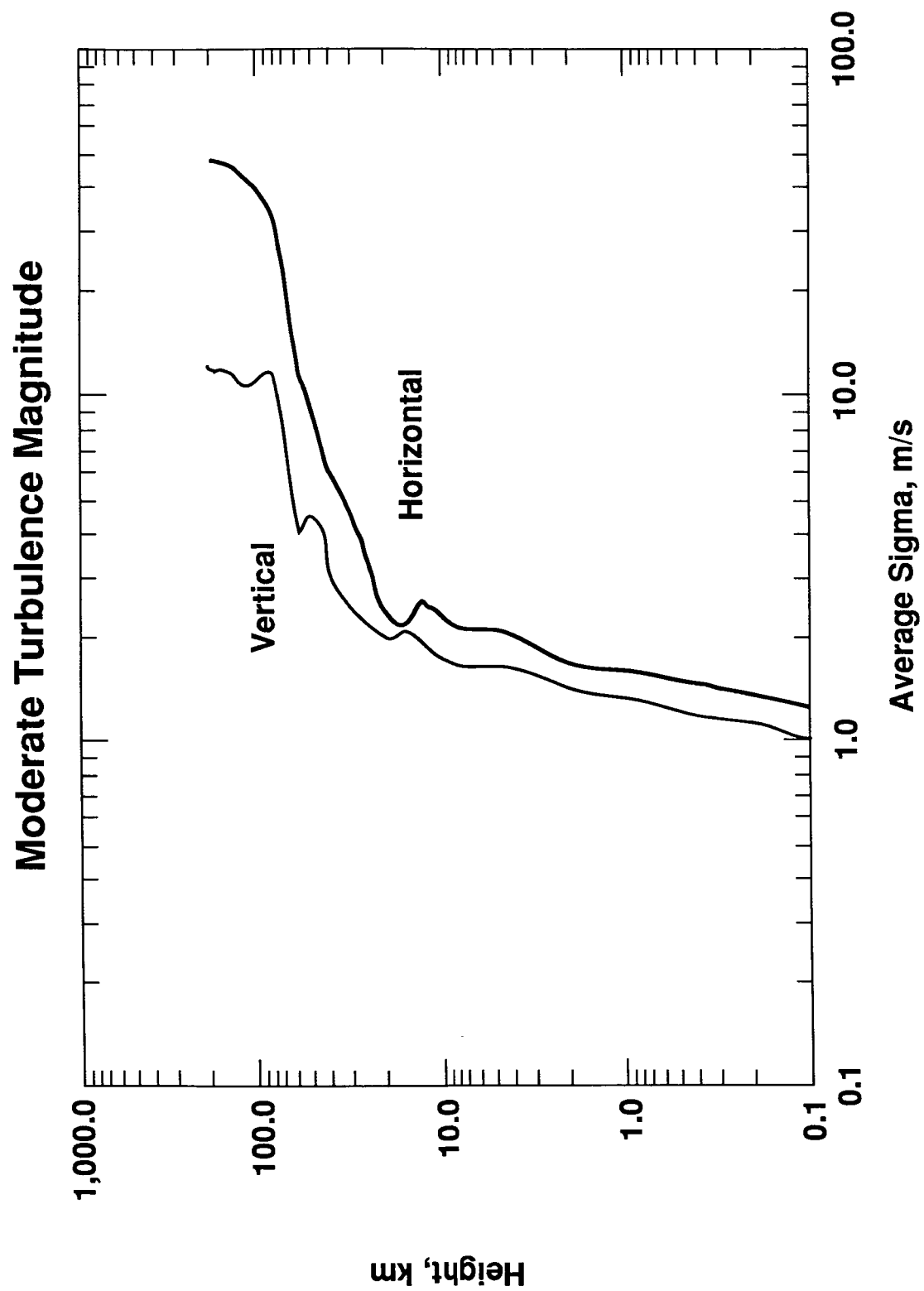


Figure 3. Horizontal and vertical magnitudes for moderate turbulence as a function of height.

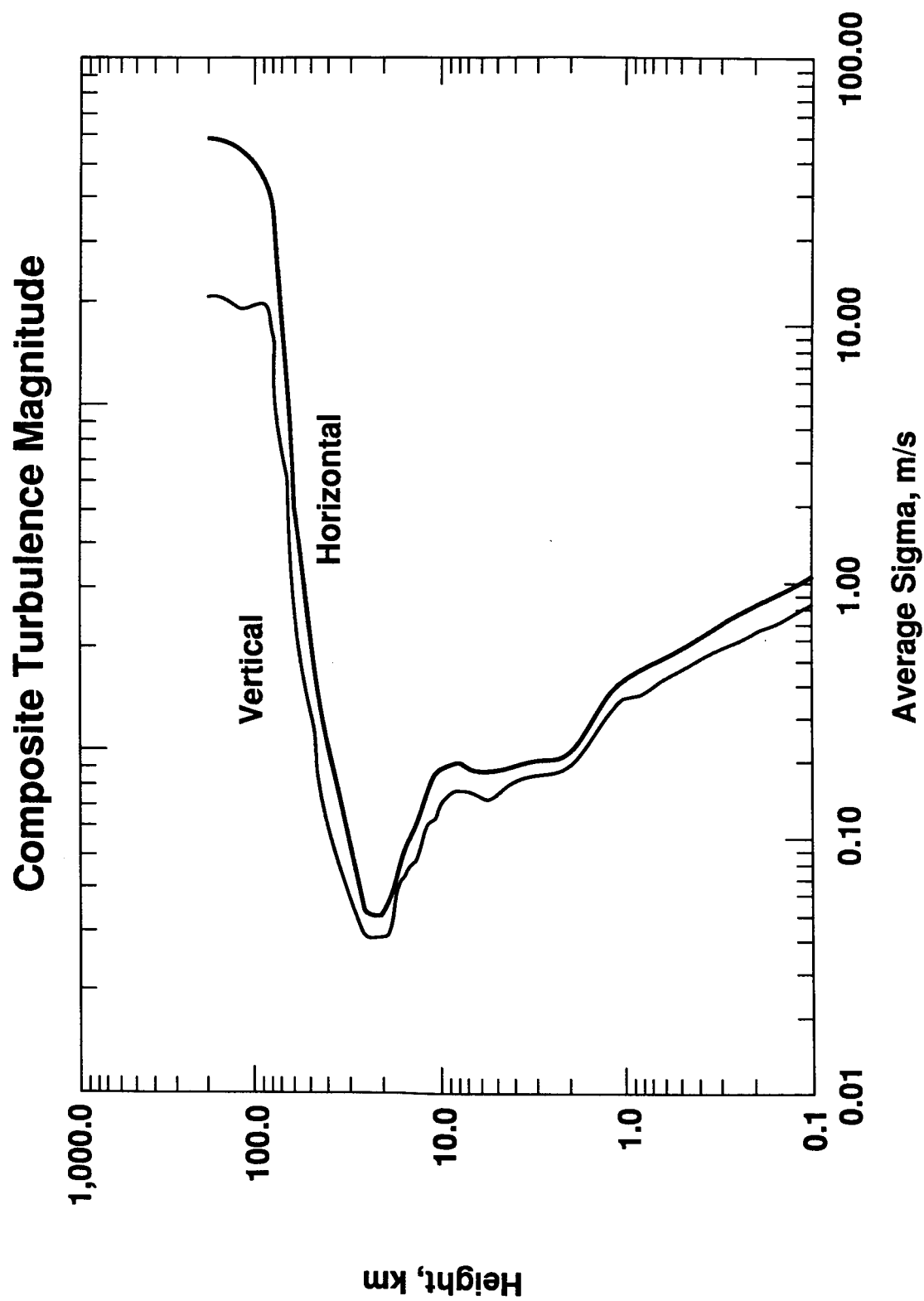


Figure 4. Horizontal and vertical magnitudes for composite turbulence (probability-weighted severe, moderate, and no turbulence cases) as a function of height.



TABLE 1. A SURVEY OF ATMOSPHERIC DISTURBANCE MODELS  
(Moorhouse and Heffley, 1986)

Model	Key Features	Sources
Dryden turbulence	A convenient spectral form based on an exponential autocorrelation function for the axial component	Dryden (1961)
von Karman turbulence	A spectral form for which the correlation function includes a finite micro-scale, thus the relative proportion of spectral power at high frequencies exceeds that of the Dryden.	von Karman (1961), Houbolt (1973)
Ornstein-Uhlenbeck turbulence	A spectral form with first-order longitudinal and transverse components	Gaonkar (1980)
Etkin one dimensional turbulence power spectra	The local turbulent velocity field is approximated by a truncated Taylor series which yields uniform and gradient components. High frequency spectral components eliminated on the basis of aircraft size. Based on Dryden form, but gradient spectra are non-realizable unless simplified.	Etkin (1961), Etkin (1959a, b), Etkin (1972)
Versine gust	A discrete gust waveform	Moorhouse and Woodcock, (1982)
Lappe low-altitude turbulence model	Experimentally-obtained data of vertical gust spectra, mean wind speed, and lapse rate were used to develop a low-level turbulence model. The turbulence spectra are presented for different types of terrain, height, and meteorological conditions.	Lappe (1966)
Multiple point source turbulence	A two-dimensional gust field generated from two or more noise sources having prescribed correlation functions and located spanwise or lengthwise on the vehicle.	Etkin (1980), Holley and Bryson (1975), Skelton (1968)

TABLE 1. (Continued)

Model	Key Features	Sources
Holley-Bryson random turbulence shaping filters	A matrix differential equation formulation of uniform and gradient components including aircraft size effects. Filter equation coefficients determined from least squares fit to multi-point-source-derived correlation functions.	Holley and Bryson (1975)
University of Washington non-Gaussian atmospheric turbulence model	Non-Gaussian model using modified Bessel functions to simulate the patchy characteristics of real-world turbulence. Spectral properties are Dryden and include gust gradients.	Reeves et al. (1974), Reeves (1969)
Delft University of Technology non-Gaussian structure of the simulated turbulent environment	Non-Gaussian model similar in form to the University of Washington model, but uses the Hilbert transformation to model intermittency as well as patchiness. Includes University of Washington model features extended to approximate transverse turbulence velocities and gradients.	van de Moesdijk (1978)
Royal Aeronautical Establishment model of non-Gaussian turbulence	Non-Gaussian turbulence model with a variable probability distribution function and a novel digital filtering technique to simulate intermittency. Spectral form approximately von Karman.	Tomlinson (1975), Jones (1976), Jewell and Heffley (1978)
The Netherlands National Aerospace Laboratory model of non-Gaussian turbulence	Similar to the Royal Aeronautical Establishment model, but extended to include patchiness and gust gradient components and transverse velocities.	Jansen (1977a, 1977b)
University of Virginia turbulence model	Models patchiness by randomizing gust variance and integral scale of basic Dryden turbulence.	Jacobson and Joshi (1977)

TABLE 1. (Continued)

Model	Key Features	Sources
MIL Standard turbulence model	First order difference equation implementation of turbulence filters based on 8785 Dryden turbulence and refitted rolling gust intensity.	Hoh et al. (1982)
Indian Institute of Science non-stationary turbulence model	Non-stationary turbulence is obtained over finite time windows by modulating a Gaussian process with either a deterministic or random process. The result is patchy-like turbulence, similar to the University of Washington model, except the time-varying statistics of turbulence are presented for the deterministic modulating functions.	Gaonkar (1980)
FAA wind shear models	Three-dimensional wind profiles for several weather system types including fronts, thunderstorms, and boundary layer. The profiles are available in table form.	Foy and Gartner (1979), Frost and Camp (1977)
STI wind shear model	Time and space domain models of mean wind and wind shear (ramp wave forms) are combined with MIL-F-8785C Dryden turbulence to obtain the total atmospheric disturbance. The magnitudes of the mean wind and wind shear are evaluated in terms of the aircraft's acceleration capabilities.	Hoh and Jewell (1976), Hefley and Jewell (1978)
Sinclair frontal wind shear model	A generic model of frontal surface wind shear derived from a reduced-order of Navier-Stokes equations. Relatively simple to use and can match the overall characteristics of measured wind shears.	Jewell et al. (1979), Sinclair and West (1978)

TABLE 1. (Continued)

Model	Key Features	Sources
MIL-F-8785B atmospheric disturbance model	Intensities and scale lengths are functions of altitude and use either Dryden or von Karman spectral forms or a one minus cosine gust. Also spectral descriptions of rotary gusts.	U.S. Dept. of Defense (1969), Chalk et al. (1969)
MIL-F-8785C atmospheric disturbance model	Same as 8785B with the addition of a logarithmic planetary boundary layer, a vector shear and a Naval carrier airwake model.	U.S. Dept. of Defense (1980)
ESDU atmospheric turbulence	Rather general, but contains comprehensive descriptive data for turbulence intensity, spectra, and probability density.	Anonymous (1974, 1975)
Boeing atmospheric disturbance model turbulence	A comprehensive model of atmospheric disturbances that includes mean wind, wind shear, and random turbulence. Turbulence is Gaussian and uses linear filters that closely approximate the von Karman spectral form. Mean wind and turbulence intensity are functions of meteorological parameters.	Barr et al. (1974)
Wasicko carrier airwake model	Includes mean wind profile, effect of ship motion, and turbulence.	Durand (1967)
Naval ship airwake model	Includes free air turbulence filters plus steady, periodic, and random components of airwake which are functions of space and time.	U.S. Dept. of Defense (1980), Nave (1978)
Vought airwake model for DD-963 class ships	Combined random and deterministic wind components for free air and ship airwake regions. Based on wind tunnel flow measurements.	Fortenbaugh (1978)

TABLE 1. (Concluded)

Model	Key Features	Sources
STI wake vortex encounter model	A two-dimensional model of the flow field due to the wake vortex of an aircraft is presented. The parameters of the flow-field model are weight, size, and speed of the vortex-generating aircraft, and distance and orientation of the vortex-encountering aircraft. Strip theory is used to model the aerodynamics of the vortex-encountering aircraft.	Johnson and Teper (1974)
Campbell and Sandborne wind shear and turbulence model	Spatial model based on joint airport weather studies (JAWS) microburst data. Permits calculation of aerodynamic loads over body of aircraft.	Campbell and Sandborne (1985)
Zhu and Etkin microburst model	Generic spatial model of microburst velocity components based on potential flow singularity distribution involving only three adjustable parameters.	Zhu and Etkin (1985)

# LISTING FOR TESTSIG PROGRAM TO TEST TURBULENCE SIMULATION MODEL FOR SIGMA VALUES AND TURBSIG SIMULATION MODEL SUBROUTINE

```

C..... PROGRAM - TESTSIG to test the TURBSIG subroutine
implicit double precision (a-h,o-z)
Open(21,File='sigmau',Status='New')
Open(22,File='sigmaw',Status='New')
Open(23,File='scaleu',Status='New')
Open(24,File='scalew',Status='New')
Open(25,File='severity',Status='New')
Open(26,File='model',Status='New')
1  Write(*,5)
5  Format(' Enter starting random number (odd integer): ')
   Read(*,*)nl
   if(nl.le.0)goto 99
   if(MOD(nl,2).ne.1)goto 1
   z = rand(nl)
   Write(*,10)
10  Format(' Enter model number 0-7: ')
   Read(*,*)model
   Write(*,15)
15  Format(' Enter starting height: ')
   Read(*,*)Z
   Call turbsig(model,z,0.,0.,sigmau,sigmaw,xlwind,zlwind,isev)
   Write(*,30)z,sigmau,sigmaw,xlwind,zlwind,isev
   Write(*,20)
20  Format(' Enter displacements DX, DZ in km: ')
   Read(*,*)dx,dz
   x = 0.
   xmax = 99999.
   if(abs(dz).le.0.)then
       Write(*,23)
23  Format(' Enter maximum x value to simulate: ')
       Read(*,*)xmax
   endif
25  Call turbsig(model,z,dx,dz,sigmau,sigmaw,xlwind,zlwind,isev)
   Write(*,30)z,sigmau,sigmaw,xlwind,zlwind,isev
   Write(21,40)z,sigmau
   Write(22,40)z,sigmaw
   Write(23,40)z,xlwind
   Write(24,40)z,zlwind
   Write(25,50)z,isev
   Write(26,30)z,sigmau,sigmaw,xlwind,zlwind,isev
30  Format(5f10.2,i3)
40  Format(2f10.2)
50  Format(f10.2,i4)
   Z = Z - abs(dz)
   x = x + abs(dx)
   if (Z.lt.0.0.or.x.gt.xmax)goto 99
   goto 25

```

99     stop  
      end

      SUBROUTINE TURBSIG(MODEL,Z,DX,DZ,SIGMAU,SIGMAW,XLWIND,ZLWIND,  
      & ISEV)

```
c
c.....
c   Simulation of turbulent wind standard deviation, sigma, selected
c   from a Rayleigh distribution, with parameters which are a
c   function of height Z.
c   Three turbulence intensities are simulated: a non-turbulent
c   background, moderate turbulence, and severe turbulence.
c   Frequencies of occurrence and minimum persistence of
c   layers are specified for moderate and severe intensities.
c   Correlation lengths for the turbulent wind field and for
c   the sigmas for the wind field are also specified
c   independently.
c   The characteristics of the Rayleigh distribution
c   for standard deviation (sigma) are determined by the expected
c   value (average value) of the sigma distribution.
c.....
c
c   Input subroutine arguments are:
c       MODEL - 0 for complete model with all specified parameters;
c               sigmas selected from Rayleigh distribution
c               1 for moderate turbulence throughout the whole run;
c               sigmas selected from Rayleigh distribution
c               2 for severe turbulence throughout the whole run;
c               sigmas selected from Rayleigh distribution
c               3 for composite turbulence (average of severe,
c               moderate and non turbulence) throughout the
c               whole run; sigmas selected from Rayleigh
c               distribution
c               4 for no turbulence throughout the whole run
c               5 for moderate turbulence of average sigma
c               throughout the whole run
c               6 for severe turbulence of average sigma
c               throughout the whole run
c               7 for composite turbulence of average sigma
c               throughout the whole run
c       Z - Current altitude in km
c       DX - Horizontal displacement since last position, in km
c       DZ - Vertical displacement since last position, in km.
c           Note - To initialize values call TURBSIG with both
c           DX = 0 and DZ = 0
c
c   Output subroutine arguments are:
c       SIGMAU - Current turbulence standard deviation for the
c               horizontal wind components, in m/s
c       SIGMAW - Current turbulence standard deviation for the
c               vertical wind component, in m/s
c       XLWIND - Current horizontal scale for turbulent wind, in km
```

```

c          ZLWIND - Current vertical scale for turbulent wind, in km
c          ISEV   - Severity parameter (1 = non turbulent,
c                  2 = moderate turbulence, 3 = severe turbulence,
c                  or 0 = composite turbulence: weighted average
c                  of categories 1-3)
c
c.....
c          implicit double precision (a-h,o-z)
c... The following static variables have values which should
c     remain unchanged from one call of the subroutine to another.
c     Some compilers require declaration of such static variables;
c     others do not.
C     Static sigmxu,sigmyu,sigmxw,sigmyw,sigsxu,sigsyu,
C     & sigsxw,sigsyw,jsev,zsev,xsev,psigu,psigw,nsev
c     double precision LSIGX(32),LSIGZ(32),LWINDX(32),LWINDZ(32),
c     & LMODZ(32),LSEVZ(32),LMODX(32),LSEVX(32),maxz(3),maxx(3)
c     dimension hgt(32),SIGMXBAR(32),SIGSXBAR(32),
c     & SIGMZBAR(32),SIGSZBAR(32),PM(32),PS(32),sigu(3),sigw(3)
c     data pi,one,two/3.14159265359d0,1.0d0,2.0d0/
c     data AFAC,BFAC/19.51615854016301d0,1.00041693941245578d0/
c... Heights for turbulence model parameters, in km
c     data hgt/0.,1.,2.,4.,6.,8.,10.,12.,14.,16.,18.,20.,25.,30.,
c     & 35.,40.,45.,50.,55.,60.,65.,70.,75.,80.,85.,90.,100.,
c     & 120.,140.,160.,180.,200./
c... Mean Value for moderate turbulence sigmas, m/s (horizontal)
c     Data SIGMXBAR/1.25,1.65,1.65,2.04,2.13,2.15,2.23,2.47,
c     & 2.62,2.44,2.21,2.26,2.71,3.73,4.59,5.26,6.22,7.27,8.7,
c     & 10.1,11.3,15.9,19.2,22.6,27.3,33.2,35.6,42.3,44.3,48.2,
c     & 48.9,49.5/
c... Mean Value for moderate turbulence sigmas, m/s (vertical)
c     Data SIGMZBAR/.98,1.36,1.43,1.68,1.69,1.69,1.73,1.79,
c     & 1.91,2.10,2.07,1.99,2.09,2.39,2.58,2.87,3.25,4.21,4.40,
c     & 4.42,4.05,5.04,6.3,8.3,10.3,11.8,11.4,10.7,10.8,11.7,
c     & 11.8,12.0/
c... Mean Value for severe turbulence sigmas, m/s (horizontal)
c     Data SIGSXBAR/3.06,3.90,4.35,6.24,7.16,7.59,7.72,7.89,
c     & 6.93,5.00,4.07,3.85,4.34,5.60,6.89,7.89,9.33,10.90,13.06,
c     & 15.1,16.9,23.8,28.7,33.8,40.9,49.8,53.3,63.4,66.4,72.2,
c     & 73.3,74.2/
c... Mean Value for severe turbulence sigmas, m/s (vertical)
c     Data SIGSZBAR/2.41,3.21,3.78,5.13,5.69,5.98,6.00,5.71,
c     & 5.05,4.31,3.81,3.38,3.34,3.59,3.87,4.30,4.88,6.31,6.60,
c     & 6.63,6.0,7.5,9.5,12.4,15.4,17.7,17.1,16.0,16.1,17.6,
c     & 17.8,18.1/
c... Horizontal scale for turbulence sigmas, in km
c     Data LSIGX/5.2,8.3,8.6,9.4,8.8,8.3,9.2,12.6,18.,21.,28.,
c     & 43.,60.,143.,177.,213.,250.,290.,330.,372.,416.,462.,
c     & 510.,555.,605.,660.,765.,1000.,1160.,1350.,1500.,1500./
c... Vertical scale for turbulence sigmas, in km
c     Data LSIGZ/3.2,6.2,7.7,8.5,8.4,8.1,8.3,11.,14.,16.,18.,
c     & 22.,33.,44.,42.,31.,26.,27.,30.,34.,38.,41.,45.,49.,
c     & 52.,56.,64.,79.,88.,100.,111.,122./
c... Horizontal scale for turbulence winds, in km
c     Data LWINDX/0.520,0.832,0.902,1.04,1.04,1.04,1.23,1.80,

```



```

& 2.82,3.40,5.00,8.64,12.0,28.6,35.4,42.6,50.1,57.9,66.0,
& 74.4,83.2,92.3,102.,111.,121.,132.,153.,200.,232.,270.,
& 300.,300./
c... Vertical scale for turbulence winds, in km
Data LWINDZ/0.323,0.624,0.831,0.972,1.01,0.98,1.10,1.54,
& 2.12,2.60,3.34,4.41,6.56,8.88,8.33,6.2,5.2,5.3,6.0,6.8,
& 7.5,8.2,9.0,9.7,10.4,11.2,12.7,15.8,17.6,20.0,22.2,24.3/
c... Probability for encountering moderate turbulence
Data PM/.867,.199,.0979,.0738,.0650,.0704,.0677,.0502,.0368,
& .0337,.0277,.0180,.0146,.0185,.0249,.0318,.0386,.0455,
& .0682,.0917,.1620,.2336,.3066,.3810,.5769,.7767,.9804,
& .9901,.9901,.9901,.9901,.9901/
c... Probability for encountering severe turbulence
Data PS/.010,.025,.0111,.0063,.0056,.0049,.0043,.0034,
& .0027,.0024,.0020,.0016,.0015,.0018,.0025,.0032,.0039,
& .0045,.0068,.0083,.0130,.0164,.0184,.0190,.0231,.0233,
& .0196,.0099,.0099,.0099,.0099,.0099/
c... Minimum vertical size for moderate turbulence layer, km
Data LMODZ/0.4,0.4,0.4,0.4,0.4,0.4,0.4,0.64,0.64,0.64,
& 0.64,0.50,0.40,0.56,0.72,0.88,1.04,1.20,1.36,1.48,1.64,
& 1.80,1.96,2.12,2.28,2.44,2.80,3.44,4.08,4.72,5.36,6.0/
c... Minimum vertical size for severe turbulence layer, km
Data LSEVZ/0.2,0.2,0.2,0.2,0.2,0.2,0.2,0.32,0.32,0.32,
& 0.32,0.25,0.20,0.28,0.36,0.44,0.52,0.60,0.68,0.74,0.82,
& 0.90,0.98,1.06,1.14,1.22,1.40,1.72,2.04,2.36,2.68,3.0/
c... Minimum horizontal size for moderate turbulence layer, km
Data LMODX/91.,74.,60.,59.,66.,69.,66.,62.,54.,38.,27.,
& 22.,21.,27.,35.,43.,50.,58.,66.,74.,88.,92.,102.,111.,
& 121.,132.,153.,200.,232.,270.,300.,300./
c... Minimum horizontal size for severe turbulence layer, km
Data LSEVX/46.,37.,30.,30.,33.,35.,33.,31.,27.,19.,14.,
& 11.,11.,14.,18.,22.,25.,29.,33.,37.,44.,46.,51.,56.,
& 61.,66.,77.,100.,116.,135.,150.,150./
c.....
if(model.gt.7.or.model.lt.0)stop ' Invalid Model Number!'
c... Non-turbulent case
if(model.eq.4)then
  SIGMAU = 0.
  SIGMAW = 0.
  XLWIND = 99999.
  ZLWIND = 99999.
  ISEV = 1
  return
endif
sqr2pi = dsqrt(two/pi)
c... Find height index for interpolation
j = 0
do 10 i = 1,32
  if(Z.lt.hgt(33-i))goto 10
  j = 33-i
  goto 20
10 continue
20 if(j.lt.1)j = 1
  if(j.gt.31)j = 31

```

```

c... Interpolate parameters on height
delz = (z - hgt(j))/(hgt(j+1) - hgt(j))
c... Average sigmas for moderate turbulence
smxb = sigmxbar(j) + delz*(sigmxbar(j+1)-sigmxbar(j))
smzb = sigmzbar(j) + delz*(sigmzbar(j+1)-sigmzbar(j))
c... Average sigmas for severe turbulence
ssxb = sigsxbar(j) + delz*(sigsxbar(j+1)-sigsxbar(j))
sszb = sigszbar(j) + delz*(sigszbar(j+1)-sigszbar(j))
c... Scales for sigmas
XLSIG = lsigx(j) + delz*(lsigx(j+1)-lsigx(j))
ZLSIG = lsigz(j) + delz*(lsigz(j+1)-lsigz(j))
c... Scales for turbulent winds
XLWIND = lwindx(j) + delz*(lwindx(j+1)-lwindx(j))
ZLWIND = lwindz(j) + delz*(lwindz(j+1)-lwindz(j))
c... Moderate turbulence case
if(model.eq.5) then
    SIGMAU = smxb
    SIGMAW = smzb
    ISEV = 2
    return
endif
c... Severe turbulence case
if(model.eq.6) then
    SIGMAU = ssxb
    SIGMAW = sszb
    ISEV = 3
    return
endif
c... Interpolate probabilities for encountering turbulence
pmz = PM(j) + delz*(PM(j+1) - PM(j))
psz = PS(j) + delz*(PS(j+1) - PS(j))
c... Composite turbulence case
if(model.eq.7) then
    SIGMAU = pmz*smxb + psz*ssxb
    SIGMAW = pmz*smzb + psz*sszb
    ISEV = 0
    return
endif
c... Minimum thickness for moderate and severe turbulence layers
maxz(2) = LMODZ(j) + delz*(LMODZ(j+1) - LMODZ(j))
maxz(3) = LSEVZ(j) + delz*(LSEVZ(j+1) - LSEVZ(j))
maxx(2) = LMODX(j) + delz*(LMODX(j+1) - LMODX(j))
maxx(3) = LSEVX(j) + delz*(LSEVX(j+1) - LSEVX(j))
C.....
c Rayleigh distribution standard deviations from mean values
smx = sqr2pi*smxb
smz = sqr2pi*smzb
ssx = sqr2pi*ssxb
ssz = sqr2pi*sszb
c... all = probability of non-turbulent severity category
all = one - (pmz+psz)
c... als = probability of less than severe category
als = one - psz
c... Use dx = 0 and dz = 0 to initialize values

```

```

        if(abs(dx).le.0.0.and.abs(dz).le.0.0)then
c..... Randomize initial sigma values
        sigmxu = rand(0)*smx
        sigmyu = rand(0)*smx
        sigmxw = rand(0)*smz
        sigmyw = rand(0)*smz
        sigsxu = rand(0)*ssx
        sigsyu = rand(0)*ssx
        sigsxw = rand(0)*ssz
        sigsyw = rand(0)*ssz
c... Calculate sigma values
        sigu(1) = 0.
        sigw(1) = 0.
        sigu(2) = dsqrt(sigmxu**2 + sigmyu**2)
        sigu(3) = dsqrt(sigsxu**2 + sigsyu**2)
        sigw(2) = dsqrt(sigmxw**2 + sigmyw**2)
        sigw(3) = dsqrt(sigsxw**2 + sigsyw**2)
c... Moderate or severe turbulence cases
        if(model.eq.1.or.model.eq.2)then
            SIGMAU = sigu(model+1)
            SIGMAW = sigw(model+1)
            ISEV = model+1
            return
        endif
c... Composite turbulence case
        if(model.eq.3)then
            SIGMAU = pmx*sigu(2) + psx*sigu(3)
            SIGMAW = pmx*sigw(2) + psx*sigw(3)
            ISEV = 0
            return
        endif
c... Select sigma values according to severity level (alpha)
        alpha = rand(0)
        if(alpha.gt.all.and.alpha.lt.als)jsev = 2
        if(alpha.le.all)jsev = 1
        if(alpha.ge.als)jsev = 3
        SIGMAU = sigu(jsev)
        SIGMAW = sigw(jsev)
c..... Store turbulence parameter values for next cycle
        psigu = sigmau
        psigw = sigmaw
        ISEV = jsev
        xsev = 0.
        nsev = 1
        return
    endif
c... Correlations for sigmas
    delx = Dsqrt((DX/XLSIG)**2 + (DZ/ZLSIG)**2)
    if(delx.lt.0.05d0) then
        rhosig = one - AFAC*delx**2
    else
        rhosig = Dexp(-BFAC*delx)
    endif
    betasig = dsqrt(one - rhosig**2)

```

### III. DIGITAL FILTER TECHNIQUES AND TURBULENCE MODELING

#### A. BACKGROUND

The currently used turbulence model for the space shuttle reentry simulation is overly conservative in that severe turbulence is assumed from reentry altitude to the Earth's surface. In the real atmosphere, turbulence is intermittent with large quiescent zones. From reentry to 10 km altitude, shuttle control is by reaction engines. Johnson Space Center (JSC) sets reaction control system redlines based on the predictions of shuttle reentry simulations. Because of the overly conservative model, the orbiter usually lands with about 270 kg (600 lb) of extra fuel. A realistic turbulence simulation model was developed to help in more rational selection of redlines. The model development was performed in two parts: a stochastic turbulence intensity model and revision of the turbulence simulation equations. This paper documents the second part of the development. Part one was performed by another contractor. The two parts are easily integrated into a single subroutine; a recommended subroutine structure is presented.

#### B. INTRODUCTION

The current turbulence model for the space shuttle reentry simulation assumes severe turbulence from reentry to the Earth's surface. This model is overly conservative because in the real atmosphere, turbulence occurs in patches embedded in large quiescent zones. Reaction engines provide control of the orbiter from reentry to about 10 km altitude. Fuel budgets for the orbiter are based on the reentry simulations with the conservative turbulence model. As a result, the orbiter lands with excess fuel. Figure 1 shows the fuel budgets and usage for a typical flight. The orbiter takes off with about 2,270 kg (5,000 lb) of fuel. About 45 kg (100 lb) of fuel is used for step-away from the external tank after separation. The amount of fuel used in on-orbit maneuvers varies and is greatest for rendezvous missions. Currently, on-orbit operations cease when fuel levels drop to the 770-kg (1,700-lb) redline. Typically, the shuttle lands with 270 kg (600 lb) of extra fuel. For some missions, this redline limits operations. For example, the Hubble Space Telescope (HST) mission needs at least 180 kg (400 lb) of additional fuel to put the HST into a higher orbit than is normally available from the shuttle.

A more realistic turbulence model will permit more rational selection of reaction control fuel redlines. The task of revising the turbulence model was performed in two parts: development of a stochastic turbulent intensity model and revision of the turbulence simulation difference equations. The first part was done by Justus [1]. The second part is the subject of this study. Part one will be discussed briefly, and part two in detail. Most derivations for part two are presented in the appendix. All difference equations and associated parameters are presented in a form appropriate for coding into flight simulation models. Each equation was tested and verified for accuracy and stability.

### C. TURBULENCE SIMULATION EQUATIONS

Turbulence is normally simulated as is shown in Figure 2. Gaussian white noise is input to the low pass filter. The output simulated turbulence is also Gaussian with the desired spectrum, e.g., von Karman. The transfer function of the filter is selected so that the desired output spectrum is obtained. If the filter is linear and the probability distribution of the input noise is Gaussian, then the output time series will also have a Gaussian probability distribution. Atmospheric turbulence is often not Gaussian. The linear filter output can be randomly modulated to obtain a more realistic output probability distribution. This study deals with generation of the Gaussian turbulence rather than with nonlinear modulation of the Gaussian time series. These refinements can be added if deemed necessary.

The most consistently observed characteristic of atmospheric turbulence is the power spectral density fall-off with frequency to the  $-5/3$  power. Only occasionally will different fall-offs be observed in turbulence spectra. The  $-5/3$  fall-off is consistent with Kolmogorov's local isotropy hypothesis and with von Karman's spectrum. The difficulty for simulators is the irrational form of the von Karman spectrum. In theory, an irrational spectrum gives rise to either an infinite order differential equation or a finite, noninteger order differential equation. Investigators have defined derivatives of noninteger order and Tatom [2] has used solutions of noninteger order differential equations to simulate irrational processes. The computational efficiency of these solutions has not yet been demonstrated.

Frequently, the rational Dryden spectrum, which falls off as frequency to the  $-2$  power, is used to simulate turbulence. Frost and Wang [3] have shown quantitative differences between simulations of fixed stick aircraft landings flying in Dryden and in von Karman turbulence. The differences are in the standard deviation of the touch-down point and in the standard deviation of aircraft sink rate. The von Karman spectrum gave lower values of these standard deviations than the Dryden model. The Dryden spectrum is, in fact, a crude approximation to the von Karman spectrum. The Frost and Wang study provides justification for looking at better approximations.

Campbell [4] approximated the von Karman spectrum with a higher order rational approximation. The discretized Campbell model was shown to have instabilities over some ranges of sampling rates. In theory, the Campbell equations were stable over all sampling rates. The instability apparently arose from finite precision computer arithmetic. This hypothesis was supported by the fact that using extended precision arithmetic delayed the onset of instability.

Mantey [5] observed a similar phenomenon and pointed out that if the difference equations were cast in modal form, the sensitivity of the equations to finite precision arithmetic instabilities was minimized. Consequently, a revision of the Campbell formulation was performed that incorporated the Mantey approach. Casting the equations in modal form permitted the analytical determination of the dependence of output turbulence standard deviation on time step. Campbell used a least squares fit of the observed standard deviation-time step curve in his original paper.

In this study, a class of transfer function approximations to the von Karman transfer function was obtained. For longitudinal spectra, approximations up to fifth order were derived. For the

remainder of this paper, the order of an approximation to the von Karman spectrum refers to the order of the approximation to the longitudinal spectrum unless the order of the transverse spectrum is specifically called out. The corresponding transverse transfer function will be one higher than the longitudinal transfer function.

The dimensionless transfer functions and corresponding spectra are presented in Figure 3. The dimensionless transfer functions can be converted to dimensional form by replacing  $s$  by  $aLs/V$  where  $L$  is the appropriate turbulent length scale,  $V$  is the velocity of the vehicle relative to the wind, and " $a$ " is von Karman's constant (1.339). From the figure, the fifth order approximation falls close to the von Karman spectrum. To better understand the closeness of the approximation, five simulations were run with the same input Gaussian white noise source data. The fifth order approximation was plotted versus the first through the fourth order approximation time series. These graphs are shown on the right in Figure 3. If the agreement was perfect, these curves would fall along a straight 45-degree line. The greater the scatter about the 45-degree line, the worse the agreement. The figure shows that the agreement becomes better as the order of the approximation increases. Similar plots were done for different sampling rates. These graphs are not shown, but the agreement becomes worse for a given approximation as the sampling rate increases.

Simulations were performed at several dimensionless sampling rates, with each of the approximations and their corresponding transverse equations, to test for instabilities. These approximations were formulated using the Advanced Continuous Simulation Language (ACSL) in single precision arithmetic. The results, presented in Tables 1 through 3, show that all simulations are stable except for the highest order and the lowest sampling rates. Second and fourth order Runge-Kutta and Euler integration were used in the simulations. Since divergence occurred for exceptionally long sampling intervals, use of even the fifth order approximation should be feasible for shuttle reentry simulations. In any case, the third order simulation was always stable and will be presented in detail.

The difference equations for the third order approximation are presented in Figure 4. The equations for both the longitudinal and transverse equations are presented. Longitudinal equations will be applied for the along wind, x-component of turbulence. The transverse equations will be used for both the y- and z-components. The equations, as shown, can be directly coded into the shuttle reentry simulation or into flight simulations for any other vehicle. Required inputs to the equations are the appropriate turbulence length scale, the appropriate relative wind velocity, turbulent intensity, and the sampling rate. The derivation of the equations is presented in the appendix. The discretization of the continuous equations was ideal in that output discrete points appear as points from the ideally sampled continuous signal. The ideal discretization is easy because of the modal form of the equations. The simulation equations currently being used at JSC do not use this ideal sampling because of the requirement for speed in a real-time simulation.

## D. INTEGRATION OF THE TURBULENCE MODEL

Justus developed a model for turbulent intensities that realistically selects turbulent intensities from a Rayleigh distribution. He provided a subroutine that offered eight options for turbulent intensities. These options range from zero turbulence to stochastic selection of moderate or severe intensities to severe turbulence at all altitudes. For the stochastic case, intensities are selected from Rayleigh distributions. Between adjacent altitude bands, the intensities are correlated. The Justus model provides all the inputs required by the turbulence equations except for the sampling rate. His model has a number of required inputs not currently used in the turbulence simulation subroutine used by JSC. The inputs required by the Justus model are shown in Figure 5. Justus developed a subroutine that calculates turbulence parameters for the x- and z-components of turbulence. The y-component is not explicitly calculated but is generated identically to the z-component. The y-component must have its own random number generator but otherwise can use coding analogous to that for the z-component. Inputs required for Justus' subroutine and not currently supplied by the JSC model are the changes in position of the vehicle from the last time step and the previous values of turbulent intensity. All of these new inputs are readily available from the current JSC model. The outputs from the model are the turbulence length scales and standard deviations required by the simulation equations. The Justus submodule and the turbulence update equations described in the previous section are essentially independent except for the Justus model outputs (length scales and standard deviations). As a result, the two efforts are easily integrated. The flow chart shown in Figure 6 describes the integration of the two elements into a revised subroutine for the JSC simulation. The turbulence equation options available are the original first order equations and the third order equations presented here.

## E. RECOMMENDED INVESTIGATIONS

A higher order, stable approximation to the von Karman spectrum and corresponding difference equations were presented in this report. The third order approximation permits simulations much closer to real turbulence than simulations based on the rational Dryden spectrum. The remaining issue is the relative importance of the higher order approximation to shuttle reentry simulation. Simulations based on the third order approximation require much more computation than first order simulations. Frost and Wang [3] give some support to the idea that better approximations are required. Running a number of simulations with the current shuttle reentry simulation using both the current turbulence model and the model with the revised turbulence equations will resolve the importance of higher order turbulence models.

The measure of importance of the higher order simulation will be the relative variability of fuel usage for the two turbulence equations. The mean fuel weight after landing will likely be about the same for both sets of simulation equations. The standard deviation about the mean will be different for the two models. The variation of fuel usage about the mean for the same mission affects the on-orbit redline selection. Each set of equations can be used to generate a probability distribution of fuel remaining at landing. If the standard deviations of the probability distributions are significantly different, use of the results from the higher order simulations are recommended for redline selection.

## REFERENCES

1. Justus, G. (1988): "A Turbulence Simulation Model for Space Shuttle Reentry Applications," Final Consulting Report for USRA, May.
2. Tatom, Frank (1988), Personal Communication, March.
3. Frost, W. and S.T. Wang (1982): "The Influence of Turbulence Models on Computer Simulated Aircraft Landing," AIAA 82-0342.
4. Campbell, C. Warren (1986): "Monte Carlo Simulation Using Approximations to the von Karman Spectra," AIAA Journal, Volume 24, Number 1, January, pp. 62-66.
5. Mantey, Patrick E. (1968): "Eigenvalue Sensitivity and State-Variable Selection," IEEE Transactions on Automatic Controls, AC-13, Number 3, June, pp. 263-268.



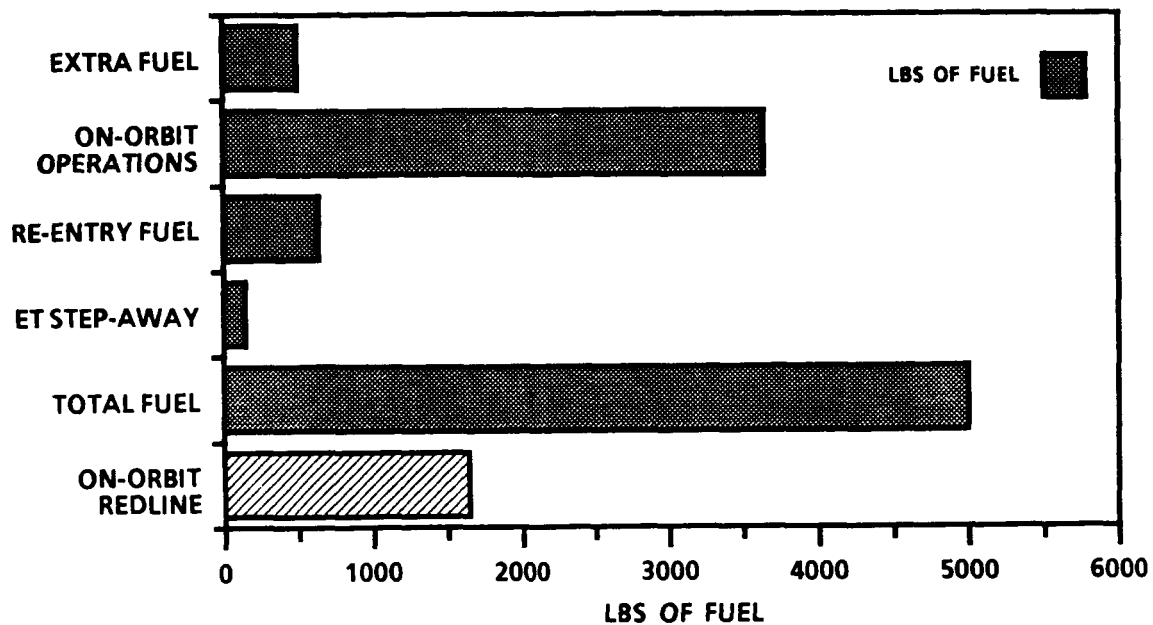


Figure 1. On-orbit maneuvering system (OMS) fuel budget.

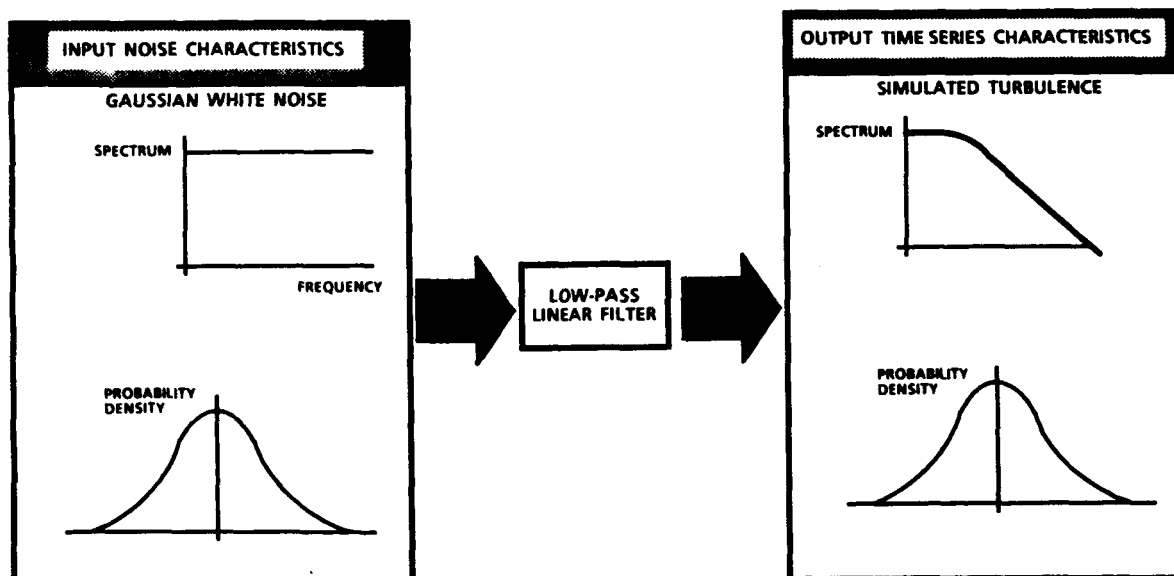


Figure 2. Gaussian turbulence simulation.

FIFTH ORDER SIMULATED  
TURBULENCE PLOTTED  
VERSUS 1st, 2nd, 3rd, AND  
4th ORDER TURBULENCE

SPECTRA

TRANSFER FUNCTIONS

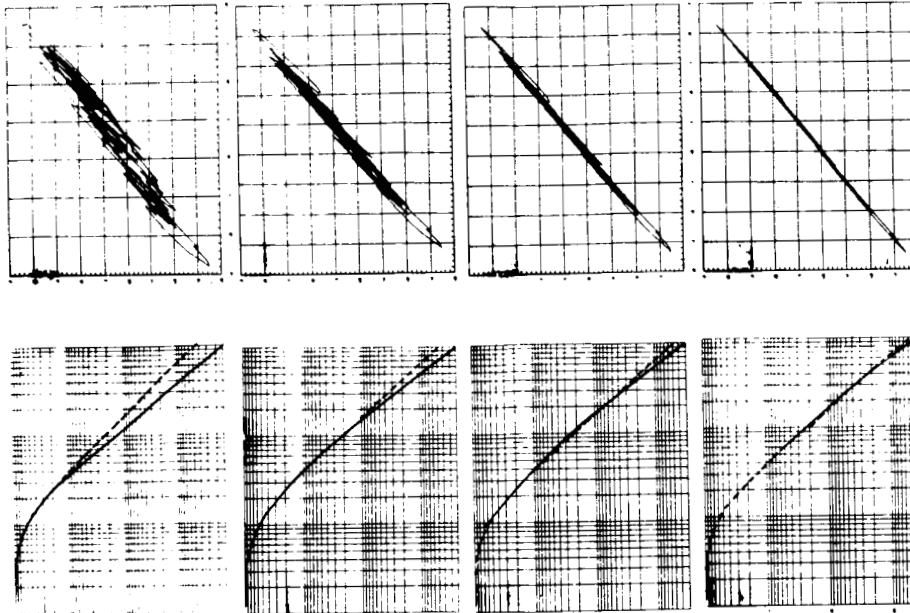
$$f_1(s) = \frac{1}{1 + \frac{5}{6}s}$$

$$f_2(s) = \frac{84s + 216}{55s^2 + 264s + 216}$$

$$f_3(s) = \frac{\frac{91}{12}s^2 + 52s + 60}{\frac{935}{216}s^3 + \frac{561}{12}s^2 + 102s + 60}$$

$$f_4(s) = \frac{24(1729s^3 + 22230s^2 + 61560s + 45360)}{21505s^4 + 412896s^3 + 1689120s^2 + 2381640s + 1088640}$$

$$f_5(s) = \frac{(259350s^4 + 5335200s^3 + 25855200s^2 + 43545600s + 23514624) / 124729}{s^5 + 30s^4 + \frac{2160}{11}s^3 + \frac{90720}{187}s^2 + \frac{2177280}{4301}s + \frac{23514624}{124729}}$$



\*SAME GAUSSIAN WHITE NOISE  
INPUTS FOR EACH ORDER OF  
SIMULATED TURBULENCE

APPROXIMATION

VON KARMAN

ORIGINAL PAGE IS  
OF POOR QUALITY

Figure 3. Performance of higher order rational turbulence models.

### DIFFERENCE EQUATION FORM

$$x_{n+1} = Gx_n + Hu_n$$

$$y_{n+1} = Cx_n$$

WHERE  $x_n$  = STATE VECTOR  
 $y_n$  = SIMULATED TURBULENT VELOCITY  
 $u_n$  = GAUSSIAN WHITE NOISE INPUT  
 $G$  = COEFFICIENT MATRIX DEFINED BELOW  
 $H$  = COLUMN VECTOR DEFINED BELOW  
 $C$  = ROW VECTOR DEFINED BELOW

### INPUTS

$\sigma_u$  = LONGITUDINAL TURBULENCE STANDARD DEVIATION  
 $L_u$  = LONGITUDINAL TURBULENT LENGTH SCALE  
 $V$  = VEHICLE VELOCITY RELATIVE TO THE WIND  
 $\tau$  = SAMPLING INTERVAL

### PARAMETERS AND CONSTANTS

$$\begin{aligned} \beta &= 1.339 L_u/V & h_{c_1} &= 0.2239020885 & c_{c_1} &= 1.751871657754 \\ \lambda_1 &= -1.02025059165 & h_{c_2} &= -0.6664166544 & c_{c_2} &= 12.0128342246 \\ \lambda_2 &= -1.6766157076134 & h_{c_3} &= 1.442514566 & c_{c_3} &= 13.86096256684 \\ \lambda_3 &= -8.1031337007321 \end{aligned}$$

$$E_i = \exp(\lambda_i \tau / \beta)$$

$$c_i = c_{c_1} + \frac{c_{c_2}}{\lambda_i} + \frac{c_{c_3}}{\lambda_i^2}$$

$$\sigma_y = \left[ \sum_{i=1}^3 \frac{c_i^2 h_{c_i}^2 (E_i - 1)^2}{\lambda_i^2 (1 - E_i^2)} + 2 \sum_{j=i+1}^3 \sum_{i=1}^2 \frac{c_i c_j h_{c_i} h_{c_j}}{\lambda_i \lambda_j} \frac{(E_i - 1)(E_j - 1)}{(1 - E_i E_j)} \right]^{\frac{1}{2}}$$

### MATRIX AND VECTOR DEFINITION

$$G = \begin{bmatrix} E_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_3 \end{bmatrix} \quad H = \frac{\sigma_u \beta}{\sigma_y} \begin{bmatrix} h_{c_1}(E_1 - 1)/\lambda_1 \\ h_{c_2}(E_2 - 1)/\lambda_2 \\ h_{c_3}(E_3 - 1)/\lambda_3 \end{bmatrix} \quad C = \frac{1}{\beta} [c_1, c_2, c_3]$$

Figure 4a. Equations for the third order longitudinal turbulence model.

### DIFFERENCE EQUATION FORM

$$x_{n+1} = Gx_n + Hu_n$$

$$y_{n+1} = Cx_n$$

WHERE  $x_n$  = STATE

$y_n$  = SIMULATED TURBULENT VELOCITY

$u_n$  = GAUSSIAN WHITE NOISE INPUT

$G$  = COEFFICIENT MATRIX DEFINED BELOW

$H$  = COLUMN VECTOR DEFINED BELOW

$C$  = ROW VECTOR DEFINED BELOW

### INPUTS

$\sigma_w$  = TRANSVERSE TURBULENCE STANDARD DEVIATION

$L_w$  = TRANSVERSE TURBULENT LENGTH SCALE

$V$  = VEHICLE VELOCITY RELATIVE TO THE WIND

$\tau$  = SAMPLING INTERVAL

### PARAMETERS AND CONSTANTS

$$\beta = 1.339 V/L_w$$

$$h_{c_1} = 11.2804723$$

$$c_{c_1} = 2.86079443756$$

$\lambda_1, \lambda_2, \text{ AND } \lambda_3$  ARE SAME

$$h_{c_2} = -1.651343027$$

$$c_{c_2} = 21.368747801$$

AS LONGITUDINAL CASE

$$h_{c_3} = 1.645595998$$

$$c_{c_3} = 34.647691313$$

$$\lambda_4 = -1.$$

$$h_{c_4} = -10.27472527$$

$$c_{c_4} = 13.860962566845$$

$$E_i = \exp(\lambda_i \tau / \beta)$$

$$c_i = c_{c_1} + \frac{c_{c_2}}{\lambda_i} + \frac{c_{c_3}}{\lambda_i^2} + \frac{c_{c_4}}{\lambda_i^3}$$

$$\sigma_y = \left[ \sum_{i=1}^4 \frac{c_i^2 h_{c_i}^2 (E_i - 1)^2}{\lambda_i^2 (1 - E_i^2)} + 2 \sum_{j=i+1}^4 \sum_{i=1}^3 \frac{c_i c_j h_{c_i} h_{c_j}}{\lambda_i \lambda_j} \frac{(E_i - 1)(E_j - 1)}{(1 - E_i E_j)} \right]^{\frac{1}{2}}$$

### MATRIX AND VECTOR DEFINITION

$$G = \begin{bmatrix} E_1 & 0 & 0 & 0 \\ 0 & E_2 & 0 & 0 \\ 0 & 0 & E_3 & 0 \\ 0 & 0 & 0 & E_4 \end{bmatrix}$$

$$H = \frac{\sigma_w \beta}{\sigma_y} \begin{bmatrix} h_{c_1}(E_1 - 1)/\lambda_1 \\ h_{c_2}(E_2 - 1)/\lambda_2 \\ h_{c_3}(E_3 - 1)/\lambda_3 \\ h_{c_4}(E_4 - 1)/\lambda_4 \end{bmatrix}$$

$$C = \frac{1}{\beta} [c_1, c_2, c_3, c_4]$$

Figure 4b. Equations for the third order transverse turbulence model.

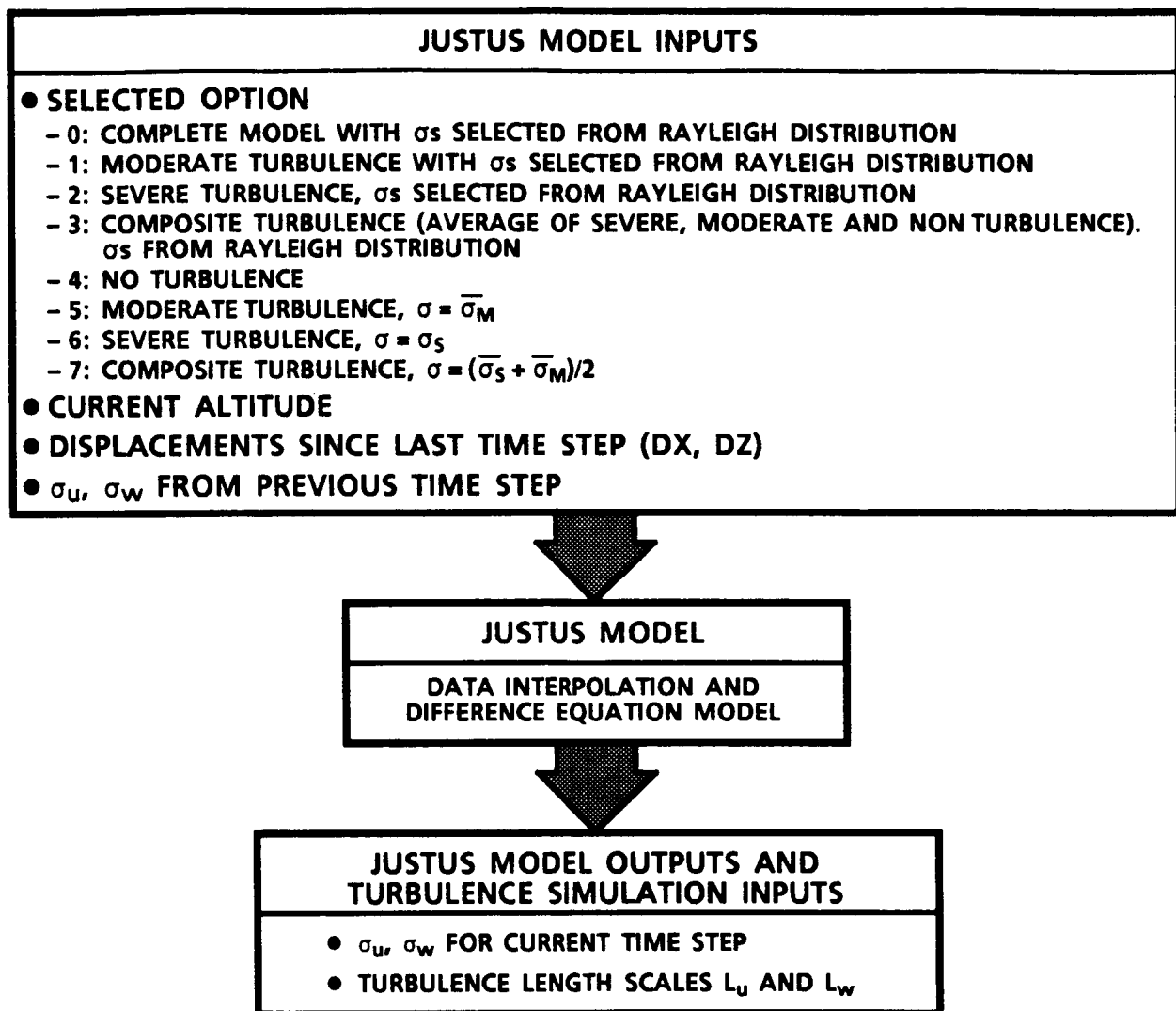


Figure 5. Characteristics of the Justus [1] turbulence model.

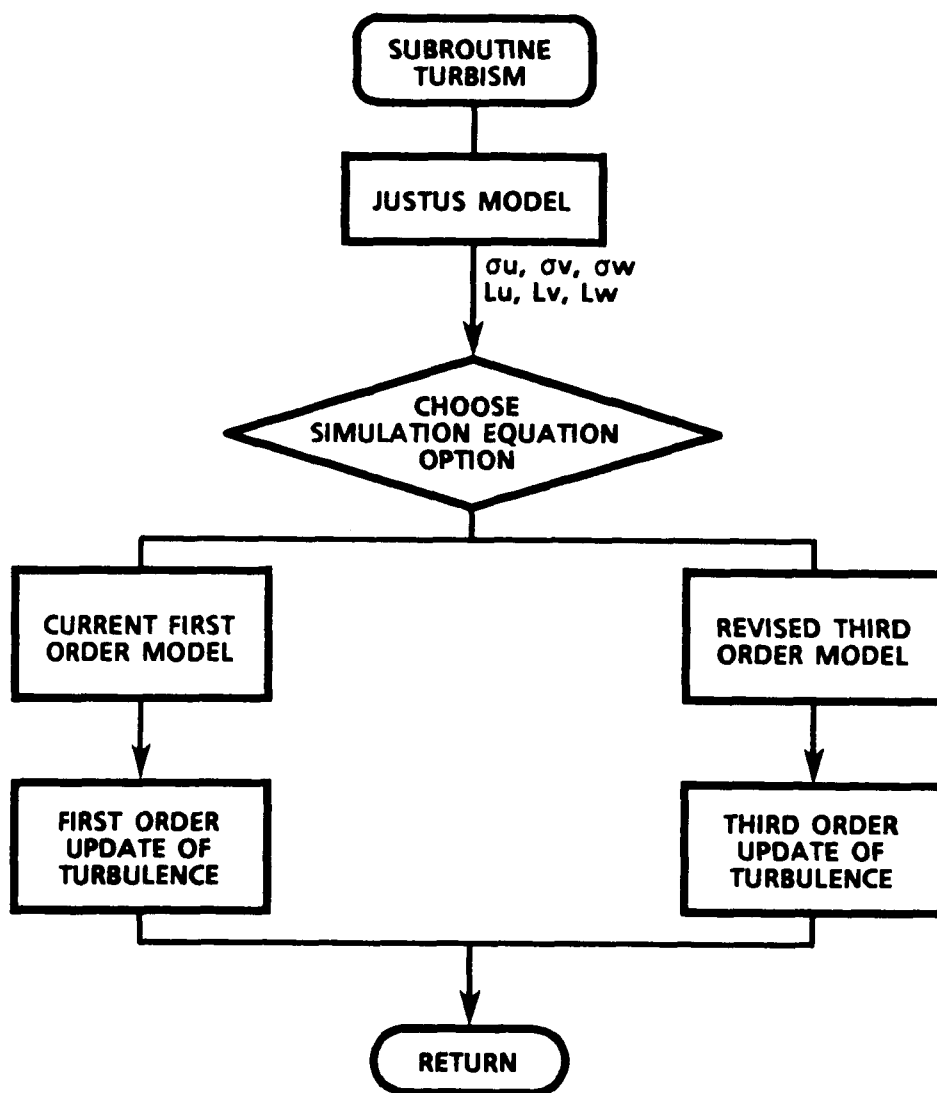


Figure 6. Recommended turbulence simulation subroutine structure.

TABLE 1. STABILITY OF EULER INTEGRATION TURBULENCE SIMULATION

TRANSFER FUNCTION ORDER	ALONG-WIND COMPONENT DIMENSIONLESS SAMPLING RATE			
	.5	.1	.01	.001
1	OK	OK	OK	OK
2	OK	OK	OK	OK
3	OK	OK	OK	OK
4	OK	OK	OK	OK
5	DIV*	OK	OK	OK
TRANVERSE COMPONENT				
2	OK	OK	OK	OK
3	OK	OK	OK	OK
4	OK	OK	OK	OK
5	OK	OK	OK	OK
6	DIV*	OK	OK	OK

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\* THESE SIMULATIONS DIVERGED

TABLE 2. STABILITY OF SECOND ORDER RUNGE KUTTA SIMULATIONS

TRANSFER FUNCTION ORDER	ALONG-WIND COMPONENT DIMENSIONLESS SAMPLING RATE			
	.5	.1	.01	.001
1	OK	OK	OK	OK
2	OK	OK	OK	OK
3	OK	OK	OK	OK
4	OK	OK	OK	OK
5	DIV*	OK	OK	OK
TRANVERSE COMPONENT				
2	OK	OK	OK	OK
3	OK	OK	OK	OK
4	OK	OK	OK	OK
5	OK	OK	OK	OK
6	DIV*	OK	OK	OK

R4-8255G-105

\* THESE SIMULATIONS DIVERGED

TABLE 3. STABILITY OF FOURTH ORDER RUNGE KUTTA TURBULENCE SIMULATIONS

TRANSFER FUNCTION ORDER	ALONG-WIND COMPONENT DIMENSIONLESS SAMPLING RATE			
	.5	.1	.01	.001
1	OK	OK	OK	OK
2	OK	OK	OK	OK
3	OK	OK	OK	OK
4	OK	OK	OK	OK
5	DIV*	OK	OK	OK
TRANSVERSE COMPONENT				
2	OK	OK	OK	OK
3	OK	OK	OK	OK
4	OK	OK	OK	OK
5	OK	OK	OK	OK
6	DIV*	OK	OK	OK

R4-8255G-106

\* THESE SIMULATIONS DIVERGED



## APPENDIX

### DERIVATION OF SIMULATION EQUATIONS FOR HIGHER ORDER APPROXIMATIONS

The turbulence simulation equations for the third order approximation in normal and state-space form are:

$$y''' + k_1 y'' + k_2 y' + k_3 y = r_1 u'' + r_2 u' + r_3 u \quad (1)$$

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \begin{bmatrix} -k_1 & -k_2 & -k_3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u \quad (2)$$

$$y = [r_1, r_2, r_3] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad (3)$$

These equations have distinct, real eigenvalues and consequently can be transformed to modal form by using the matrix whose columns are eigenvectors of the coefficient matrix. For a matrix in the above form, the appropriate transformation is given by:

$$T = \begin{bmatrix} 1 & 1 & 1 \\ 1/\lambda_1 & 1/\lambda_2 & 1/\lambda_3 \\ 1/\lambda_1^2 & 1/\lambda_2^2 & 1/\lambda_3^2 \end{bmatrix} \quad (4)$$

where  $\lambda_i$  = eigenvalues.

The resulting equations are in the form  $\dot{x} = Ax + Bu$  where  $A$  is the following diagonal matrix.

$$A = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & -\lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \quad (5)$$

The matrix B is given by  $B = T^{-1}U$  where  $U = [1 \ 0 \ 0]^T$ . The superscript T indicates the column vector transpose of the row vector  $[1 \ 0 \ 0]$ . The eigenvalues in equation (5) are in dimensionless form. To convert them to dimensional form, the eigenvalues must be divided by  $1.339 L/V$ , L is the turbulent length scale and V is the vehicle velocity relative to the wind. To perform a simulation, the above equations must be discretized. The discretization chosen is in the following form.

$$x_{n+1} = Gx_n + Hfu_n,$$

where

f = a factor to ensure the correct standard deviation of y

$$G = e^{A\tau}$$

$$H = A^{-1}(e^{A\tau} - I)B$$

$\tau$  = sampling interval, and

I = 3x3 identity matrix.

Once the difference equation is obtained, the variance of the output y is required. For equations in the  $x = Gx + Hu$  form, the covariance of the state vector is given by the expected value of  $x^T x = P$ . P is given by the solution to the Lyapunov equation

$$P = GPG^T + HH^T.$$

The unknown quantity in this equation is a matrix. Because G is a diagonal matrix, P can be determined easily.

$$P_{ij} = \frac{h_i h_j}{1 - e^{\lambda_i \tau / \beta} e^{\lambda_j \tau / \beta}}$$

where  $h_i$  is the  $i$ th component of  $H$ .

Once  $P$  is known, the covariance of  $y$  can be determined from:

$$\sigma_y^2 = CPC^T$$

The preceding equation is expanded in Figure 4. The above approach can be applied for the equations of any order. The general equation is:

$$\sigma_y^2 = \sum_{i=1}^n \frac{c_i^2 h_{ci}^2 (E_i - 1)^2}{\lambda_i^2 (1 - E_i^2)} + 2 \sum_{j=i+1}^n \sum_{i=1}^{n-1} \frac{c_i c_j h_{ci} h_{cj} (E_i - 1)(E_j - 1)}{\lambda_i \lambda_j (1 - E_i E_j)}$$

All parameters in the above equation are presented in Figure 4.

Derivations in this appendix were specifically for the third order equations; the higher order equations are in identical form. Only the eigenvalues and the constants are different for different equations.



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